LENDI INSTITUTE OF ENGINEERING AND TECHNOLOGY

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EMWTL Problems

Unit-1

S. No	Question
1	Calculate the electric force on a 10nC charge located at (0, 3, 1) point. Point charges 1mC and -2mC are located at (3, 2,-1) and (-1,-1, 4) respectively. (5M, CO1, Apply)
2	(i) Find the force on a 1 nC point charge located at $\mathbf{a}_x - 3\mathbf{a}_y + 1\mathbf{a}_z$. (ii) Find the electric field E at $\mathbf{a}_x - 3\mathbf{a}_y + 1\mathbf{a}_z$. Point charges 5nC and -2nC are located at $2\mathbf{a}_x + 4\mathbf{a}_z$ and $-3\mathbf{a}_x + 5\mathbf{a}_z$ respectively. (5M, CO1, Understand)
3	(i) Find D at (2, 90^0, 0). (ii) Calculate the work done in moving a 10µC charge from point A (1, 30^0, 120^0) to B (4, 90^0, 60^0). Given the potential $V=(10/r^2)\,\mathrm{Sin}\Theta\mathrm{Cos}\varphi$ (5M, CO1, Understand)
4	Find the volume charge density at any point in space when the electric flux density is given as $\mathbf{D}=x^4\mathbf{a}_x - x^3y\,\mathbf{a}_z$ (5M, CO1, Understand)
5	Point charges Q1 and Q2 are respectively located at (4, 0,-3) and (2, 0, 1).If Q2 = 4nC, find Q1 such that. i. The **E** at (5, 0,6) has no Z-component. ii. The force on a test charge at (5, 0, 6) has no X-component. (5M, CO1, Understand)
6	Find the total current I, When Apply Gauss's law to derive the boundary conditions at a conductor-dielectric interface. In a cylindrical conductor of radius 2mm, the current density varies with distance from the axis according to $J = 10.3\ A/m^2$ (5M, CO1, Understand)
7	Find the energy in the system, when three point charges -1 nC, 4 nC and 3 nC are located at (0, 0, 0), (0, 0, 1) and (1, 0, 0) respectively. (5M, CO1, Understand)
8	Find the potential at (1, 0, 1). Assuming '0' potential at infinity. Two point charges - 4µC and 5µC are located at (2, -1, 3) and (0, 4, -2) respectively. (5M, CO1, Understand)
9	Find the potential at point (2, 3, -4) of a line charge ρ_L= 400 pC/m lies along the X-axis. The surface of zero potential passes through the point P (0, 5, 12). (5M, CO1, Understand)
10	A parallel plate capacitance has 500mm side plates of square shape separated by 10mm distance. A sulphur slab of 6mm thickness with $\in r = 4$ is kept on the lower plate find the capacitance of the set-

	up. If a voltage of 100 volts is applied across the capac (5M, CO1, Understand)
11	Find the capacitance of a parallel plate capacitor with dielectric, mica filled between plates. The relative permittivity ($\in_r$) of mica is 6. The plates of the capacitor are square in shape with 0.254 cm side. Separation between the two plates is 0.254 cm. (5M, CO1, Understand)
12	Find the total charge on the sheet, when the finite sheet 0 < x < 1, 0 < y < 1 on the Z=0 plane has a charge $\rho_s = xy(x^2 + y^2 + 25)^{\frac{3}{2}}$ nC/m^3 (5M, CO1, Understand)
13	Find the current passing through hemispherical shell of radius 20 cm, if **J** = $\frac{1}{r^3}(2cos\theta\, \overline{a_r} + sin\theta\, \overline{a_\theta})$A/m^2 (5M, CO1, Understand)
14	Find the force on a charge of -100mC located at P (2, 0, 5) in free space due to another charge 300µC located at Q (1, 2, 3). (5M, CO1, Understand)
15	Find the electric field strength, E at The origin, a charge of -0.3mC is located at A (25, -30, 15) cm and a second charge of 0.5mC is located at B (-10, 8, 12) cm. (5M, CO1, Remember)

Unit-2

S. No	Question
1	Find **H** at (0, 0, 4), when a Circular loop located on $x^2 + y^2 = 9$, z = 0 carries current of 10A along a_φ (5M, CO2, Understand)
2	**E**=10Sin(ωt-βz)a_yV/M in free space, determine **D**, **B**, **H**. (5M, CO2, Understand)
3	What is the magnetic field, H in Cartesian coordinates due to z-directed current element? Find J if I = 2A. (5M, CO2, Understand)
4	Find **H** at P (2, 3, 4) if there is a current filament on the z-axis carrying 8 mA in the $\mathbf{a_z}$ direction. Obtain an expression for the magnetic field intensity due to an infinite current element directed along origin. (5M, CO2, Understand)
5	Find the conduction current density at point (2, 0, -1) meters and Find the current enclosed by a square loop y=1, 0<x<1, 0<z<1.In a conducting medium the magnetic field is given as, **H**= $y^2z\, \mathbf{a_x} + 2(x+1)yz\, \mathbf{a_y} - (x+1)\, z^2\, \mathbf{a_z}$. (5M, CO2, Understand)
6	Calculate the magnitude of the magnetic flux density at (1, 1, 1). In a current free region of relative permeability 1, the magnetic scalar potential is given as $V_m=x^2y+ y^2x+z$. (5M, CO2, Understand)
7	Determine the total flux around the coil in a circular coil of radius 2.0 cm is in a magnetic flux density of 10wb/m^2, if the plane of the coil is perpendicular to the field. (5M, CO2, Understand)
8	Determine **B** and displacement current density. In free space D = D_mSin (wt +β z) $\mathbf{a_x}$. (5M, CO2, Understand)
9	Calculate **D**, $\mathbf{J_D}$. In free space **E**=cos(ωt-50x)a_y V/m

	(5M, CO2, Understand)
10	Find the electric flux density and volume charge density if The electric field $E=X^2a_x$ Whose in a medium $\varepsilon_r=2$. (5M, CO2, Understand)
11	Find **H** at (0, 0, -1) cm of a thin ring of radius 5 cm is placed on plane z =1 cm so that its center is at (0, 0, 1) cm. If the ring carries 50 mA along a_ϕ
12	Find the current density **J** in the both the regions (r<a and r>a) of a long straight conductor with radius 'a' has a magnetic field strength $H = \frac{Ir}{2\pi a^2}\boldsymbol{a}_\phi$ within the conductor (r<a) and $H = \frac{I}{2\pi r}\boldsymbol{a}_\phi$ outside the conductor (r>a). (5M, CO2, Understand)
13	Determine the magnetic flux for the surface described by (i) ρ =1 m., $0 \le \phi \le \frac{\pi}{2}$ (ii) a sphere of radius 2 m. If the magnetic field is of the form $H = \frac{1}{\rho}cos\phi\, \boldsymbol{a}_\rho$ A/m (5M, CO2, Understand)
14	Find the flux density at the center of a square loop of 10 turns carrying a current of 10 A. The loop is in air and has a side of 2 m. (5M, CO2, Understand)
15	B) Find **B** at (-1, 2, 5) of a current distribution gives rise to the vector magnetic potential $\mathbf{A} = x^2y\boldsymbol{a}_x + y^2x\boldsymbol{a}_y - 4xyz\boldsymbol{a}_z$ Wb/m (5M, CO2, Understand)

Unit-3

S. No	Question
1	Determine the propagation constant and intrinsic impedance of the medium, if $\varepsilon_0 = 9$, $\mu = \mu_0$ for the medium in which a wave with a frequency f=0.3 GHz is propagating when σ=0. (5M, CO3, Apply)
2	Calculate the frequency and permittivity of the medium when The wavelength of an x-directed plane wave in a lossless medium is 0.25 m and the velocity of propagation is 1.5 X 1010 cm/s. The wave has z-directed electric field with an amplitude equal to 10 (5M, CO3, Apply)
3	Calculate the Phase constant, Phase Velocity and Equation for H_Z, if $E_Z = H_Y=0$, for a uniform plane wave travelling in a X-Direction in the free space $E_Y=10Sin(2\pi 10^8 t-\beta x)$. (5M, CO3, Apply)
4	Calculate the following the average power if the magnetic field intensity **(H)** has a magnitude of 5 mA/meter in a medium defined by $\epsilon_r = 4, \mu_r = 1$. (5M, CO3, Apply)
5	Calculate the i) Velocity of Propagation ii) Intrinsic impedance in free space an in Material iii) Wavelength in free space in material, when f=1GHz, an EM Wave Propagated through a material μ_r =5 and $\varepsilon_r = 10$. (5M, CO3, Apply)

MID- 2

Unit-3

SNO	Question
6	Determine the (i) magnitude of the electric field for the plane wave in free space (ii) magnitude of the electric field when the wave propagates in a medium which is characterised by σ = 0, $\mu = \mu_0$

	and $\epsilon = 4\varepsilon_0$ When the amplitude of the magnetic field in a plane wave is 2 A/m (5M, CO3, Apply)
7	Calculate the Depth of penetration, of an EM wave in copper at f = 60 Hz and f = 100 MHz for copper σ = 5.8 X 10^7 mho/m $\epsilon_r = 1, \mu_r = 1$ (5M, CO3, Apply)
8	Determine the angle by which **E** tilts as the wave crosses the boundary when an EM wave travelling in free space incidents on a dielectric medium with relative constant is 2 at an angle 45^0. (5M, CO3, Apply)
9	(i) Determine the Phase Velocity of the wave (ii) Amplitude and Direction of magnetic field intensity If the electric field E has only a 'y' component with amplitude of 20 V/m. A plane wave travelling in +ve X-direction in a loss less unbounded medium hav (5M, CO3, Apply)
10	Determine Maximum Energy Density in a medium defined by $\epsilon_r = 4, \mu_r = 1$, when given an electric field **E** is 942.5mV/meter, and magnetic field **H** is 5 mA/meter. (5M, CO3, Apply)
11	Determine the average power density absorbed by copper if a plane wave with a frequency of 2 MHz is incident upon a copper conductor normally. The wave has an electric field amplitude of **E** = 2 mV/m. Copper has $\epsilon_r = 1, \mu_r = 1$ and σ= 5.8 X 10^7 Ω/m (5M, CO3, Apply)
12	Determine (i) The Intrinsic Impedance, (ii) the Propagation Constant, (iii) the Phase Velocity for a Dry ground has a conductivity of 5 X 10^{-4} mhos/m and relative dielectric constant of 10 at a frequency of 500 MHz. (5M, CO3, Apply)
13	Determine the displacement current density and conduction current density if the same field exists in a medium whose conductivity is given by 2.0X103mho/cm and electric field strength of a radio broadcast signal at a TV receiver is given by E = 5.0cos(wt (5M, CO3, Apply)
14	Determine the magnitude of the electric field. When the amplitude of a magnetic field in free space is 10mA/m. (5M, CO3, Apply)
15	Determine **B, H, D** for an electric field in a medium which is source-free is given by **E** = 1.5cos(10^8t – βz)$\mathbf{a}_x$ V/m, where E_m is the amplitude, ω is the angular frequency and β is the phase constant. (5M, CO3, Apply)

Unit-4

S. NO	Question
1	Find the inductance and capacitance of the line if a transmission line with air as dielectric has Z_0 = 50Ω and a phase constant of 3.0 rad/m at 10MHz. (5M, CO4, Remember)
2	Find R, L, G, C and wavelength at 0.1 GHz. If a transmission line in which no distortion is present has the following parameters Z_0=50 Ω α = 0.02m^{-1}, v=0.6v_0 (5M, CO4, Remember)
3	Find the characteristic impedance of the line when a copper coaxial line has an outside tubing of thickness 1.8 mm and its outside diameter is 30mm. The thickness of the inner tubing is 1.0mm

	and its outside diameter is 8 mm. (5M, CO4, Remember)
4	Find the characteristic impedance for sections of a line 10 metre long and 500 metre long, if lossless transmission line used in a TV receiver has a capacitance of 50pF /m and an inductance of 200 nH/m. (5M, CO4, Remember)
5	(i) Find the time delay for a cable 1 m long, (ii) propagation velocity, and (iii) propagation delay over a cable length of 10 m. If a signal of 30 MHz is transmitted through a coaxial cable which has a capacitance of 30 pF/m and an inductance of 500 nH/m (5M, CO4, Remember)
6	Find the Characteristics Impedance and Propagation Constant of the line when A high frequency line has the following primary constants L=1.2mH/Km, C=0.05μF/Km. R = G = negligible. (5M, CO4, Remember)
7	B) Find the Phase Constant and the Reflection coefficient of a line of length 50 m. Characteristic Impedance Z_0 is 50Ω and wavelength on the line is 0.45 m of lossless transmission line is terminated in a load impedance of 30-j23Ω. (5M, CO4, Remember)
8	Find the Propagation Constant and Intrinsic Impedance of the medium when $\sigma = 0$. If $\varepsilon_r = 9$, $\mu = \mu_0$ for the medium in which a wave with a frequency of f = 0.3 GHz is propagating. (5M, CO4, Remember)
9	Find out i) Propagation velocity ii) Phase constant at an operating frequency of 100 KHz iii. Characteristic impedance of the line when a lossless transmission line of length 100m has an inductance of 28 μH and capacitance of 20nF. (5M, CO4, Remember)
10	Find out the characteristic impedance of the line when a two-wire open air line, whose diameter is 2.588 mm, is used in several applications and the wires are spaced at 290 mm between the centres. (5M, CO4, Remember)
11	Find the Capacitance C per unit length A lossless transmission line has a characteristic impedance of $Z_O = 75$ Ω and inductance L = 300 nH/m. (5M, CO4, Understand)
12	Find the phase constant β of given a transmission line has a propagation velocity $V_P = 2 \times 10^8$ m/s and operate at frequency f = 100 MHz. (5M, CO4,Remember)
13	Find the normalised impedance magnitude for a transmission line which is terminated in a normalised Impedance Z_n, VSWR = 2. (5M, CO4, Remember)
14	Find the inductance per unit length L and capacitance per unit length C of a lossless transmission line has characteristic impedance Z_0 is 60Ω and a propagation velocity V_P is 2.5×10^8 m/s. (5M, CO4,Remember)
15	Find the inductance and capacitance of the transmission line with air as dielectric has Z_0=50W and a phase constant of 3.0 rad/m at 10 MHz. (5M, CO4,Remember)

Unit-5

S. NO	Question
1	Calculate the reflection coefficient and standing wave ratio if a low transmission line of 100Ω characteristic impedance is connected to a load of 300Ω

	(5M, CO5, Apply)
2	Determine the input impedance and corresponding reflection coefficient value at 25λ away from load if a transmission line of characteristics impedance 50 ohms operating at 1GHz is connected to the load impedance of 50+j75 ohms (5M, CO5, Apply)
3	Determine the input impedance of 75 Ω lossless transmission line of length 0.1λ when the load is a short. (5M, CO5, Apply)
4	Determine the reflection coefficient and VSWR if a transmission line of 2000Ω characteristic impedance is connected to a load of 800Ω (5M, CO5, Apply)
5	Analyze the reflection coefficient and standing wave ratio. A low transmission line of 100Ω characteristic impedance is connected to a load of 300Ω. (5M, CO5, Analyze)
6	Determine the centre-to-centre spacing in air for a transmission line with a characteristic impedance of 300 Ω, designed to feed a 90 Ω load using a quarter-wave transformer. The line is constructed with conductors in the form of tubes, each having a diam (5M, CO5, Understand)
7	Determine the VSWR, Reflection Coefficient and Input Impedance of a transmission line which is connected to a load impedance of ZL = (100 - J40)Ω. The characteristic impedance of the line is 50 Ω then the operating frequency is 3MHz and length is 2.5cm (U (5M, CO5, Apply)
8	Analyze L and C values at a frequency of 80 MHz, a lossless transmission line has Z_0=300Ω and wave length of 2.5m. (5M, CO5, Analyze)
9	Design a stub to match a transmission line which is connected to a load impedance of Z_L = (450 - J600) Ω. The characteristic impedance of the line is 300 Ω then the operating frequency is 20MHz (Using Smith Chart) (5M, CO5, Apply)
10	Discover the position and the length of a series open-circuit stub required to match the 50 Ω transmission line is connected to a cellular phone antenna with load impedance Z_L = (100 + J80) Ω (Using Smith Chart) (5M, CO5, Analyze)
11	Inspect effective area of a Hertzian dipole operating at100MHz. (5M, CO5, Analyze)
12	Determine the Voltage Reflection Coefficient, Voltage Standing Wave Ratio and Input Impedance of the line of a transmission line of length 0.40λ has a characteristic impedance of 100Ω and is terminated in a load impedance of 200 + J180Ω (Using Smith Chart (5M, CO5, Apply)
13	Determine VSWR, Reflection Coefficient and Find distance from the load to the first voltage minimum of transmission line with a characteristic impedance Z0 = 50Ω is connected to load impedance Z_L = (120+j90)Ω (Using Smith Chart) (5M, CO5, Apply)
14	Determine VSWR, Reflection Coefficient and input impedance at 0.15λ distance toward the generator of transmission line with a characteristic impedance Z_0 = 60Ω is connected to load impedance Z_L = (30-j40)Ω (Using Smith Chart) (5M, CO5, Apply)
15	Determine Reflection Coefficient, Voltage Standing Wave Ratio and input impedance at 0.2λ from the load of transmission line with a characteristic impedance Z_0 = 75Ω is connected to load impedance Z_L = (50+j25) Ω (Using Smith Chart) (5M, CO5, Apply)

Problems Solutions Document

UNIT-1

1) Calculate the electric force on a 10nC charge located at (0, 3, 1) point. Point charges 1mC and -2mC are located at (3, 2,-1) and (-1,-1, 4) respectively.

Solution:

$$\bar{F} = \frac{Q}{4\pi \in_0} \sum_{K=1}^{2} Q_K \frac{\bar{r} - \bar{r}_K}{\left|\bar{r} - \bar{r}_K\right|^3}$$

$$= \frac{10\times10^{-9}}{4\pi \in_0}\left[1\times10^{-3}\frac{\left(-3\bar{a}_x + \bar{a}_y + 2\bar{a}_z\right)}{\left(\sqrt{9+1+4}\right)^3} - 2\times10^{-3}\frac{\left(\bar{a}_x + 4\bar{a}_y - 3\bar{a}_z\right)}{\left(\sqrt{1+16+9}\right)^3}\right]$$

$$\because \quad \in_0 = 8.854\times10^{-12} \text{ and } \pi = 3.14$$

$$= 90\left[\frac{\left(-3\bar{a}_x + \bar{a}_y + 2\bar{a}_z\right)\times10^{-3}}{52.38} - 10^{-3}\frac{\left(2\bar{a}_x + 8\bar{a}_y - 6\bar{a}_z\right)}{132.57}\right]$$

$$= 90\times10^{-3}\left[\bar{a}_x\left(\frac{-3}{52.38} - \frac{2}{132.57}\right) + \bar{a}_y\left(\frac{1}{52.38} - \frac{8}{132.57}\right) + \bar{a}_z\left(\frac{2}{52.38} + \frac{6}{132.57}\right)\right]$$

$$= 90\times10^{-3}\left[-0.0723\bar{a}_x - 0.0413\bar{a}_y + 0.0834\bar{a}_z\right]$$

$$= -0.0065\,\bar{a}_x - 0.0037\,\bar{a}_y + 0.0075\,\bar{a}_z \text{ N.}$$

2) (i) Find the force on a 1 nC point charge located at $\mathbf{a}_x$- 3$\mathbf{a}_y$ + 1$\mathbf{a}_z$. (ii) Find the electric field E at $\mathbf{a}_x$ - 3$\mathbf{a}_y$ + 1$\mathbf{a}_z$. Point charges 5nC and -2nC are located at 2$\mathbf{a}_x$ + 4$\mathbf{a}_z$ and -3$\mathbf{a}_x$ + 5$\mathbf{a}_z$ respectively.

(a) We know

$$\bar{F} = \frac{Q}{4\pi \in_0} \sum_{K=1}^{2} Q_K \frac{\bar{r} - \bar{r}_K}{\left|\bar{r} - \bar{r}_K\right|^3}$$

$$= 10^{-9}\times9\times10^{9}\times10^{-9}\left[5\frac{\left(-\bar{a}_x - 3\bar{a}_y + 3\bar{a}_z\right)}{\left(\sqrt{1+9+9}\right)^3} - \frac{2\left(4\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z\right)}{\left(\sqrt{16+9+4}\right)^3}\right]$$

$$= 9\times10^{-9}\left[\bar{a}_x\left(\frac{-5}{82.81} - \frac{8}{156.169}\right) + \bar{a}_y\left(\frac{-15}{82.81} + \frac{6}{156.169}\right) + \bar{a}_z\left(\frac{15}{82.81} - \frac{4}{156.169}\right)\right]$$

$$= 9\times10^{-9}\left[\bar{a}_x(-0.112) + \bar{a}_y(-0.143) + \bar{a}_z(0.155)\right]$$

$$= -1.008\bar{a}_x - 1.287\bar{a}_y + 1.395\bar{a}_z \text{ nN}$$

(b) $\bar{E} = \dfrac{\bar{F}}{Q}$, here Q = 1 nC

$$\therefore \quad \bar{E} = -1.008\bar{a}_x - 1.287\bar{a}_y + 1.395\bar{a}_z \text{ V/m}$$

3) (i) Find D at (2, 90^0, 0). (ii) Calculate the work done in moving a 10µC charge from point A (1, 30^0, 120^0) to B (4, 90^0, 60^0). Given the potential $V=(10/r^2)\ \mathrm{Sin}\Theta\mathrm{Cos}\varphi$

$$\overline{E} = -\nabla V$$

Since V is given in spherical co-ordinate system, consider ∇V in spherical co-ordinate system

$$\therefore\ \overline{E} = -\frac{\partial v}{\partial r}\overline{a}_r + \frac{1}{r}\frac{\partial v}{\partial \theta}\overline{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial v}{\partial \phi}\overline{a}_\phi$$

$$= -\left(10\left(-2r^{-3}\right)\sin\theta\cos\phi\,\overline{a}_r + \frac{1}{r}\frac{10\cos\theta\cos\phi}{r^2}\overline{a}_\theta + \frac{1}{r\sin\theta}\frac{10\sin\theta(-\sin\phi)}{r^2}\overline{a}_\phi\right)$$

$$= -\left(10\left(-2r^{-3}\right)\sin\theta\cos\phi\,\overline{a}_r + \frac{1}{r}\frac{10\cos\theta\cos\phi}{r^2}\overline{a}_\theta + \frac{1}{r\sin\theta}\frac{10\sin\theta(-\sin\phi)}{r^2}\overline{a}_\phi\right)$$

$$= \left(\frac{20\sin\theta\cos\phi}{r^3}\overline{a}_r + \frac{-10\cos\theta\cos\phi}{r^3}\overline{a}_\theta + \frac{10\sin\phi}{r^3}\overline{a}_\phi\right)$$

$$= \frac{10}{r^3}\left(2\sin\theta\cos\phi\,\overline{a}_r - \cos\theta\cos\phi\,\overline{a}_\theta + \sin\phi\,\overline{a}_\phi\right)$$

$$\overline{D} = \overline{E}\in_0$$

$$= \frac{8.825\times10^{-11}}{r^3}\left[2\sin\theta\cos\phi\,\overline{a}_r - \cos\theta\cos\phi\,\overline{a}_\theta + \sin\phi\,\overline{a}_\phi\right]$$

$$= \frac{8.825\times10^{-11}}{r^3}\left[2.1.1\overline{a}_r - 0 + 0\right]$$

$$\overline{D}(2, \frac{\pi}{2}, 0) = 22.1\,\overline{a}_r\ \text{pC/m}^2$$

(b) Work done $= -Q\int_A^B \overline{E}\cdot d\overline{L} = -Q\left(-V_{AB}\right)$

$$= Q\,(V_B - V_A)$$

$$V_B = \frac{10}{16}1.\frac{1}{2} = 0.3125\ V$$

$$V_A = \frac{10}{1}\frac{1}{2}(-0.5) = -5\times0.5 = -2.5\ \text{V}$$

$$V_B - V_A = 2.8125\ \text{V}$$

$$\text{W} = 10^{-3}\times10\times(V_B - V_A) = 28.125\ \text{mJ}$$

4) Find the volume charge density at any point in space when the electric flux density is given as $D = x^4\mathbf{a_x} - x^3y\,\mathbf{a_z}$

$$\vec{D} = x^4\vec{a}_x - x^3y\vec{a}_z$$

$$\rho_v = \nabla \cdot \vec{D}$$

$$\rho_v = \frac{\partial}{\partial x}(x^4) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(-x^3y)$$

$$\rho_v = 4x^3 + 0 + 0$$

$$\boxed{\rho_v = 4x^3\ \mathrm{C/m^3}}$$

5) Point charges Q1 and Q2 are respectively located at (4, 0,-3) and (2, 0, 1).If Q2 = 4nC, find Q1 such that. i. The **E** at (5, 0,6) has no Z-component. ii. The force on a test charge at (5, 0, 6) has no X-component.

Solution

We have $\bar{F} = \dfrac{Q}{4\pi \in_0}\sum_{K=1}^{2} Q_K \dfrac{\bar{r}-\bar{r}_K}{|\bar{r}-\bar{r}_K|^3}$

(a) $$\bar{E} = \frac{\bar{F}}{Q} = \frac{1}{4\pi \in_0}\left[\frac{Q_1|(5,0,6)-(4,0,-3)|}{\left(\sqrt{1+81}\right)^3} + \frac{4\times10^{-9}|(5,0,6)-(2,0,1)|}{\left(\sqrt{9+25}\right)^3}\right]$$

Given $\bar{E}$ has no Z – component, considering only Z components on both sides

$$0 = \frac{1}{4\pi \in_0}\left[\frac{Q_1\times 9}{\left(\sqrt{82}\right)^3} + \frac{4\times10^{-9}\times 5}{\left(\sqrt{34}\right)^3}\right]$$

$$\frac{Q_1\times 9}{\left(\sqrt{82}\right)^3} = -\frac{4\times10^{-9}\times 5}{\left(\sqrt{34}\right)^3}$$

$$Q_1 = -\frac{20}{9}\left(\sqrt{\frac{41}{17}}\right)^3 nC = -8.3\,\mathrm{nC}$$

(b) Given the force on test charge has no X-component

$$0 = \frac{Q}{4\pi \in_0}\left[\frac{Q_1}{\left(\sqrt{82}\right)^3} + \frac{4\times10^{-9}\times 3}{\left(\sqrt{34}\right)^3}\right]$$

$$= \frac{Q_1}{\left(\sqrt{82}\right)^3} = -\frac{4\times10^{-9}\times 3}{\left(\sqrt{34}\right)^3}$$

$$Q_1 = -12\left(\sqrt{\frac{41}{17}}\right)^3 \mathrm{nC} = -44.95\,\mathrm{nC}$$

6) Find the total current I, When Apply Gauss's law to derive the boundary conditions at a conductor-dielectric interface. In a cylindrical conductor of radius 2mm, the current density varies with distance from the axis according to J = 10.3 A/m^2

Solution:

$$A = \pi r^2 = \pi(2 \times 10^{-3})^2 = \pi \times 4 \times 10^{-6} = 4\pi \times 10^{-6}\,\mathrm{m}^2$$

$$I = 10.3 \times 4\pi \times 10^{-6}$$

$$I = 41.2\pi \times 10^{-6}\,\mathrm{A}$$

$$I \approx 129.43 \times 10^{-6}\,\mathrm{A}$$

$$\boxed{I \approx 129.43\,\mu\mathrm{A}}$$

7) Find the energy in the system, when three point charges -1 nC, 4 nC and 3 nC are located at (0, 0, 0), (0, 0, 1) and (1, 0, 0) respectively.

$$W_E = W_1 + W_2 + W_3$$

$$= 0 + Q_2V_{21} + Q_3(V_{31} + V_{32})$$

$$= Q_2 \cdot \frac{Q_1}{4\pi \in_0 |r_2 - r_1|} + \frac{Q_3}{4\pi \in_0}\left[\frac{Q_1}{|r_3 - r_1|} + \frac{Q_2}{|r_3 - r_2|}\right]$$

$$= \frac{1}{4\pi \in_0}\left(Q_1Q_2 + Q_1Q_3 + \frac{Q_2Q_3}{\sqrt{2}}\right)$$

$$= \frac{1}{4\pi . \dfrac{10^{-9}}{36\pi}}\left(-4 - 3 + \frac{12}{\sqrt{2}}\right).10^{-18}$$

$$= 9\left(\frac{12}{\sqrt{2}} - 7\right)\,\mathrm{nJ} = 13.37\,\mathrm{nJ}$$

8) Find the potential at (1, 0, 1). Assuming '0' potential at infinity. Two point charges - 4μC and 5μC are located at (2, -1, 3) and (0, 4, -2) respectively.

Solution

$$V = \frac{Q_1}{4\pi \in_0 |r - r_1|} + \frac{Q_2}{4\pi \in_0 |r - r_2|}$$

$$V = \frac{-4 \times 10^{-6}}{4\pi \in_0 |(1,0,1) - (2,-1,3)|} + \frac{5 \times 10^{-6}}{4\pi \in_0 |(1,0,1) - (0,4,-2)|}$$

Simplifying, we get

$$V = -5.872\ \mathrm{kV}$$

9) Find the potential at point (2, 3, -4) A line charge ρ_L= 400 pC/m lies along the X-axis. The surface of zero potential passes through the point P (0, 5, 12).

The general equation for the potential difference due to a line charge along the X-axis is:

$$V_{AB} = -\frac{\rho_L}{2\pi\epsilon_0}\int_{R_A}^{R_B}\frac{dL}{R}$$

- R is the radial distance from the line charge to the points A and B,
- dL is the differential length along the line charge,
- ρ_L is the line charge density,
- $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$ is the permittivity of free space.

Distance R_A from the line charge to point $A(0, 5, 12)$:

$$R_A = \sqrt{0^2 + 5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13\,\mathrm{m}$$

- **Distance** R_B from the line charge to point $B(2, 3, -4)$:

$$R_B = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{29} \approx 5.39\,\mathrm{m}$$

Formula for Potential Difference: The potential difference between points A and B due to the line charge is given by: $V_{AB} = -\frac{\rho_L}{2\pi\epsilon_0}\ln\left(\frac{R_A}{R_B}\right)$

Substitute Known Values: $V_{AB} = -\frac{400 \times 10^{-12}}{2\pi(8.854 \times 10^{-12})}\ln\left(\frac{13}{5.39}\right)$

Calculating the logarithmic term: $\ln\left(\frac{13}{5.39}\right) = \ln(2.411) \approx 0.878$

$$\frac{400 \times 10^{-12}}{2\pi(8.854 \times 10^{-12})} \approx 7.2$$

$$V_{AB} = -7.2 \times 0.878 = -6.31\,\mathrm{V}$$

10) A parallel plate capacitance has 500mm side plates of square shape separated by 10mm distance. A sulphur slab of 6mm thickness with $\epsilon_r = 4$ is kept on the lower plate find the capacitance of the set-up. If a voltage of 100 volts is applied across the capacitor, calculate the voltages at both the regions of the capacitor between the plates

Area of parallel plates, $A = 500 \text{ mm} \times 500 \text{ mm} = 500 \times 500 \times 10^{-6} \text{ m}^2$.

Distance of separation $d = 10 \text{ mm} = 10 \times 10^{-3}$ m.

Thickness of sulphur slab $d_2 = 6 \text{ mm} = 6 \times 10^{-3}$ m.

Relative permittivity of sulphur slab $\in_r = 4$.

Voltage applied across the capacitor V = 100 V.

Here the capacitor has two dielectric media,

One medium is the sulphur slab of thickness (d_2)6 mm,

since the distance between the plates(d) is 10 mm

The remaining distance is air $d_1 = d - d_2 = 4$ mm.

The other dielectric medium is air with thickness (d_1) 4 mm.

The capacitance of the parallel plate capacitor with two dielectric media is

$$C = \frac{\in_0 A}{\left(\frac{d_1}{\in_{r1}} + \frac{d_2}{\in_{r2}}\right)} \text{ F}$$

Here $\quad \in_{r1}$ (air) $= 1$, $\in_{r2} = \in_r = 4$

$$C = \frac{8.854\times10^{-12}\times500\times500\times10^{-6}}{\left(\frac{4\times10^{-3}}{1}+\frac{6\times10^{-3}}{4}\right)} = 0.402\ \text{nF}$$

The charge $Q = CV = 0.402 \times 10^{-9} \times 100 = 4.02 \times 10^{-8}$ C

The value of capacitance (C_1) in delectric-1 i.e., air is

$$C_1 = \frac{\in_0 A}{d_1} = \frac{8.854\times10^{-12}\times500\times500\times10^{-6}}{4\times10^{-3}} = 0.55\ \text{nF}$$

The value of capacitance (C_1) in delectric-1 i.e., air is

$$C_1 = \frac{\in_0 A}{d_1} = \frac{8.854\times10^{-12}\times500\times500\times10^{-6}}{4\times10^{-3}} = 0.55\ \text{nF}$$

Similarly, The value of capacitance (C_2) in delectric-2 i.e., sulphur is

$$C_2 = \frac{\in A}{d_2} = \frac{4\times8.854\times10^{-12}\times500\times500\times10^{-6}}{6\times10^{-3}} = 1.48\ \text{nF}$$

We have $V = V_1 + V_2$

Where V_1 is the voltage at the region of the capacitor plate near dielectric-1 i.e., air.

and V_2 is the voltage at the region of the capacitor plate near dielectric-2 i.e., sulphur.

$$V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1} = \frac{4.02\times10^{-8}}{0.55\times10^{-9}} = 73.1\ \text{V}$$

$\therefore \quad V_2 = 100 - 73.1 = 26.9$ V

11) Find the capacitance of a parallel plate capacitor with a dielectric, mica filled between plates. Relative permittivity ($\in_r$) of mica is 6. The plates of the capacitor are square in shape with 0.254 cm side. Separation between the two plates is 0.254 cm.

We have $\quad C = \dfrac{\in A}{d}$

Here $\quad \in = \in_0 \in_r = 8.854\times10^{-12}\times6$

$$C = \frac{8.854\times10^{-12}\times6\times0.254\times0.254\times10^{-4}}{0.254\times10^{-2}} = 0.1349\ \text{pF}$$

12) Find the total charge on the sheet, when the finite sheet 0 < x < 1, 0 < y < 1 on the Z=0 plane has a charge $\rho_s = xy(x^2+y^2+25)^{\frac{3}{2}}$ nC/m^3

Solution

(a) $dQ = \rho_s\, ds$

$$Q = \int_s \rho_s ds$$

$$= \int_{x=0}^{1}\int_{y=0}^{1} xy\left(x^2+y^2+25\right)^{3/2} n\, dx\, dy$$

$$= n\int_{x=0}^{1} x\int_{y=0}^{1}\left(x^2+y^2+25\right)^{3/2}\frac{1}{2}d(y^2)dx$$

$$= n\int_{x=0}^{1} x\left[\left(x^2+y^2+25\right)^{5/2}\right]_0^1 \frac{2}{5}\frac{1}{2}dx$$

$$= \frac{n}{5}\int_{x=0}^{1}\left[\left(x^2+26\right)^{5/2}-\left(x^2+25\right)^{5/2}\right]\frac{1}{2}d(x^2)$$

$$= \frac{n}{5}\left[\left(x^2+26\right)^{7/2}-\left(x^2+25\right)^{7/2}\right]_0^1\frac{1}{7}$$

$$= \frac{n}{35}\left[(27)^{7/2}-2(26)^{7/2}+(25)^{7/2}\right]$$

$$= \frac{n}{35}[102275.868136-179240.733942+78125]$$

$Q = 33.15$ nC

13) Find the current passing through hemispherical shell of radius 20 cm, if $\mathbf{J} = \frac{1}{r^3}(2cos\theta\, \overline{a_r} + sin\theta\, \overline{a_\theta})$A/m^2

Solution

$$I = \int \bar{J} . d\bar{s}$$

Since it is sphere $d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$

(a) $\phi = 0$ to 2π, $\theta = 0$ to $\pi/2$ and $r = 0.2$ m for hemispherical shell

$$I = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \frac{1}{r^3} \left(2\cos\theta \, \bar{a}_r + \sin\theta \, \bar{a}_\theta \right) \cdot r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r a$$

$$= \frac{1}{r} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} 2\cos\theta \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{r} \int_{\theta=0}^{2\pi} \int_{\theta=0}^{\pi/2} \sin 2\theta \, d\theta \, d\phi$$

$$= \frac{1}{r} \int_{\phi=0}^{2\pi} \left[\frac{-\cos 2\theta}{2} \right]_0^{\pi/2} d\phi$$

$$= -\frac{1}{2r} (-1-1)(2\pi) = \frac{2\pi}{0.2} = 10\pi = 31.4A$$

14) Find the force on a charge of -100mC located at P (2, 0, 5) in free space due to another charge 300μC located at Q (1, 2, 3).

Solution:

$$\mathbf{R}_{21} = \mathbf{R_1} - \mathbf{R_2}$$

$$P(2,0,5) \Rightarrow \mathbf{R_1} = 2\hat{i} + 5\hat{k}$$

$$Q(1,2,3) \Rightarrow \mathbf{R_2} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\mathbf{R}_{21} = (2\hat{i} + 5\hat{k}) - (1\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$|\mathbf{R}_{21}| = \sqrt{(1)^2 + (-2)^2 + (2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3\,\mathrm{m}$$

$$\hat{r} = \frac{\mathbf{R}_{21}}{|\mathbf{R}_{21}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$$\mathbf{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{|\mathbf{R}_{21}|^2} \hat{r}$$

Where ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$).

$$\mathbf{F}_{21} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} \times \frac{(-100 \times 10^{-3})(300 \times 10^{-6})}{3^2} \times \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$$\mathbf{F}_{21} = \frac{-100 \times 300 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times 9} \times \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\mathbf{F}_{21} = \frac{-30000 \times 10^{-12}}{4\pi \times 8.854 \times 10^{-12} \times 9} \times \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\mathbf{F}_{21} = \frac{-30000}{4\pi \times 8.854 \times 9} \times \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\mathbf{F}_{21} \approx -3 \times 10^4 \times \left(\frac{1}{36\pi}\right) \times \left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\mathbf{F}_{21} = -3 \times 10^4 \times \left(\frac{1}{36\pi}\right) \times \left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\mathbf{F}_{21} \approx -3 \times 10^4 \times \left(\hat{i} - 2\hat{j} + 2\hat{k}\right) / 36$$

15) Find the electric field strength, E at The origin, a charge of -0.3mC is located at A (25, -30, 15) cm and a second charge of 0.5mC is located at B (-10, 8, 12) cm.

Solution:

$$Q_1 = -0.3\,\text{mC} = -0.3 \times 10^{-3}\,\text{C at } A(25, -30, 15)\,\text{cm} = (0.25, -0.30, 0.15)\,\text{m}$$

$$Q_2 = 0.5\,\text{mC} = 0.5 \times 10^{-3}\,\text{C at } B(-10, 8, 12)\,\text{cm} = (-0.10, 0.08, 0.12)\,\text{m}$$

Distance from the Origin:

For Q_1:

$$\vec{R}_1 = (0 - 0.25, 0 - (-0.30), 0 - 0.15) = (-0.25, 0.30, -0.15)$$

$$|\vec{R}_1|^2 = 0.25^2 + 0.30^2 + 0.15^2 = 0.0625 + 0.09 + 0.0225 = 0.175$$

$$|\vec{R}_1|^3 = (0.175)^{3/2}$$

For Q_2:

$$\vec{R}_2 = (0.10, -0.08, -0.12)$$

$$|\vec{R}_2|^2 = 0.01 + 0.0064 + 0.0144 = 0.0308$$

$$|\vec{R}_2|^3 = (0.0308)^{3/2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-0.3 \times 10^{-3}}{(0.175)^{3/2}} \cdot (-0.25, 0.30, -0.15) + \frac{0.5 \times 10^{-3}}{(0.0308)^{3/2}} \cdot (0.10, -0.08, -0.12)\right]$$

Then,

$$\vec{E} = 9 \times 10^9\left[\frac{-0.3 \times 10^{-3}}{(0.175)^{3/2}} \cdot (-0.25, 0.30, -0.15) + \frac{0.5 \times 10^{-3}}{(0.0308)^{3/2}} \cdot (0.10, -0.08, -0.12)\right]$$

UNIT-2

1) Find **H** at (0, 0, 4), when a Circular loop located on $x^2 + y^2 = 9$, z = 0 carries current of 10A along a_φ

Solution

Here $\rho = 3$, h = 4 and I = 10 A

$$\therefore \quad \overline{H} = \frac{10}{4\pi} \frac{3^2}{\left(3^2 + 4^2\right)^{3/2}} \overline{a}_z \int_0^{2\pi} d\phi$$

$$= 0.36\overline{a}_z \text{ A/m}$$

Similarly $\overline{H}_{(0,0,-4)} = \overline{H}_{(0,0,4)} = 0.36\overline{a}_z$ A/m

2) **E**=10Sin(ωt-βz)a_yV/M in free space, determine **D**, **B**, **H**.

Given:

$$\vec{E} = 10\sin(\omega t - \beta z)\,\hat{a}_y$$

Medium: **Free space**

In free space:

- $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
- $\mu_0 = 4\pi \times 10^{-7}$ H/m
- $\eta_0 = \sqrt{\mu_0/\varepsilon_0} \approx 377\ \Omega$ (intrinsic impedance of free space)

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{D} = 8.854 \times 10^{-12} \cdot 10\sin(\omega t - \beta z)\,\hat{a}_y = 8.854 \times 10^{-11}\sin(\omega t - \beta z)\,\hat{a}_y \text{ C/m}^2$$

$$\vec{H} = \frac{E}{\eta_0}\hat{a}_x = \frac{10}{377}\sin(\omega t - \beta z)\,\hat{a}_x \approx 0.0265\sin(\omega t - \beta z)\,\hat{a}_x \text{ A/m}$$

$$\vec{B} = \mu_0\vec{H} = 4\pi \times 10^{-7} \cdot 0.0265\sin(\omega t - \beta z)\,\hat{a}_x \approx 3.33 \times 10^{-9}\sin(\omega t - \beta z)\,\hat{a}_x \text{ T}$$

3) What is the magnetic field, H in Cartesian coordinates due to z-directed current element? Find J if I = 2A.

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

Where:

- $\rho = \sqrt{x^2 + y^2}$: radial distance from wire
- $\hat{a}_\phi$: tangential unit vector in the x-y plane

$\hat{a}_\phi = -\dfrac{y}{\rho}\,\hat{a}_x + \dfrac{x}{\rho}\,\hat{a}_y$

$$\vec{H}(x,y) = \frac{I}{2\pi\rho}\left(-\frac{y}{\rho}\hat{a}_x + \frac{x}{\rho}\hat{a}_y\right) = \frac{I}{2\pi\rho^2}\left(-y\,\hat{a}_x + x\,\hat{a}_y\right)$$

$\rho = \sqrt{x^2 + y^2}$, and $I = 2\ \mathrm{A}$:

$$\boxed{\vec{H}(x,y) = \frac{1}{\pi(x^2+y^2)}\left(-y\,\hat{a}_x + x\,\hat{a}_y\right)\ \mathrm{A/m}}$$

$$\vec{J} = \nabla \times \vec{H}$$

4) Find **H** at P (2, 3, 4) if there is a current filament on the z-axis carrying 8 mA in the $\mathbf{a_z}$ direction. Obtain an expression for the magnetic field intensity due to an infinite current element directed along origin.

$$\vec{H} = \int_{-\infty}^{\infty} \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3}$$

Vector Setup:

- $d\vec{l} = dz\,\hat{a}_z$
- $\vec{R} = \vec{r} - \vec{r}' = (2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z) - (0\hat{a}_x + 0\hat{a}_y + z\hat{a}_z) = 2\hat{a}_x + 3\hat{a}_y + (4-z)\hat{a}_z$

Cross Product:

$$d\vec{l} \times \vec{R} = dz\,\hat{a}_z \times (2\hat{a}_x + 3\hat{a}_y + (4-z)\hat{a}_z) = dz(2\hat{a}_y - 3\hat{a}_x)$$

Integral:

$$\vec{H} = \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{(2\hat{a}_y - 3\hat{a}_x)\,dz}{[(z-4)^2 + 2^2 + 3^2]^{3/2}} = \frac{I}{4\pi}(2\hat{a}_y - 3\hat{a}_x)\int_{-\infty}^{\infty} \frac{dz}{[z^2+13]^{3/2}}$$

Integral Evaluation: Standard result:

$$\int_{-\infty}^{\infty} \frac{dz}{(z^2+a^2)^{3/2}} = \frac{2}{a^2} \quad \text{for } a = \sqrt{13}$$

$$\int_{-\infty}^{\infty} \frac{dz}{[z^2+13]^{3/2}} = \frac{2}{13}$$

Final Expression: $\vec{H} = \dfrac{I}{4\pi}(2\hat{a}_y - 3\hat{a}_x)\ \dfrac{2}{13} = \dfrac{I}{26\pi}(2\hat{a}_y - 3\hat{a}_x)$

Substitute $I = 8\,\mathrm{mA}$:

$$\vec{H} = \frac{8\times 10^{-3}}{26\pi}(2\hat{a}_y - 3\hat{a}_x) \approx -294\hat{a}_x + 196\hat{a}_y\ \mu\mathrm{A/m}$$

5) Find the conduction current density at point (2, 0, -1) meters and Find the current enclosed by a square loop y=1, 0<x<1, 0<z<1.In a conducting medium the magnetic field is given as, **H**= y^2z $\mathbf{a_x}$ + $2(x+1)yz$ $\mathbf{a_y}$ – $(x+1)\, z^2$ $\mathbf{a_z}$.

We have current density $\bar{J} = \nabla \times \bar{H}$

$$\bar{J} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z & 2(x+1)yz & -(x+1)z^2 \end{vmatrix}$$

$$\bar{J} = -2(x+1)y\bar{a}_x + (z^2+y^2)\bar{a}_y\ \mathrm{A/m^2}$$

$$\bar{J}_{(1,0,-3)} = 9\bar{a}_y\ \mathrm{A/m^2}$$

Current is $I = \int_s \bar{J}.\bar{ds}$, here $\bar{ds} = dxdz\bar{a}_y$

$$\therefore\ I = \int_s \left(-2(x+1)y\bar{a}_x + (z^2+y^2)\bar{a}_y\right)dx\, dz\, \bar{a}_y = \int_{x=0}^{1} dx \int_{z=0}^{1} (z^2+y^2)dz = 1.33\ \mathrm{A}$$

6) Calculate the magnitude of the magnetic flux density at (1, 1, 1). In a current free region of relative permeability 1, the magnetic scalar potential is given as $V_m = x^2y + y^2x + z$.

$$H = -\nabla V_m$$

Given, $V_m = x^2y + y^2x + z$

$$\Rightarrow\ H = -\left[\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right](x^2y + y^2x + z)$$

$$H = -(2xy + y^2)a_x - (x^2 + 2xy)a_y - (1)a_z$$

$$H(1,1,1) = -3a_x - 3a_y - a_z$$

magnitude of magnetic flux density ,

$$B = \mu_0 \mu_r H$$

$$(\because \mu_0 = 4\pi \times 10^{-7})$$

$$= 4\pi \times 10^{-7} \times 1 \times \sqrt{3^2+3^2+1^2}$$

$$= 4\pi \times 10^{-7} \times \sqrt{19}$$

$$\therefore \ B = 5.5 \mu T$$

7) Determine the total flux around the coil in a circular coil of radius 2.0 cm is in a magnetic flux density of 10wb/m^2, if the plane of the coil is perpendicular to the field.

Solution The flux density = 10 wb/m^2

Area of the coil, $S = \pi r^2$

$$= \pi \times (2.0 \times 10^{-2})^2$$

$$= 4\pi \times 10^{-4}$$

Total flux, $\phi = \mathbf{B} \cdot \mathbf{S}$

As **B** and **S** are in the same direction,

$$\mathbf{B} \cdot \mathbf{S} = BS$$

$$\phi = 10 \times 4\pi \times 10^{-4}$$

$$\boxed{\phi = 12.56 \text{ mwb}}$$

8) Determine **B** and displacement current density. In free space D = D_mSin (wt +β z) $\mathbf{a_x}$.

From the equations in the image: $\mathbf{D} = \epsilon_0 \mathbf{E}$

Thus, the electric field $\mathbf{E}$ is: $\mathbf{E} = \frac{1}{\epsilon_0}\mathbf{D}$

Substitute the given expression for $\mathbf{D}$

$$\mathbf{E} = \frac{1}{\epsilon_0} D_m \sin(\omega t + \beta z)\hat{a}_x$$

Maxwell's third equation is: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

$$\mathbf{E} = \frac{1}{\epsilon_0} D_m \sin(\omega t + \beta z)\hat{a}_x$$

Now, calculate the curl of $\mathbf{E}$. Since $\mathbf{E}$ only has a component along the $\hat{a}_x$ direction, the curl simplifies

$$\nabla \times \mathbf{E} = \hat{a}_z \frac{\partial}{\partial y}\left(\frac{1}{\epsilon_0} D_m \sin(\omega t + \beta z)\right) = 0$$

Thus, the curl of $\mathbf{E}$ is zero, and we can conclude that $\mathbf{B}$ is related to $\mathbf{E}$ by the equation:

$$\mathbf{B} = \frac{1}{\epsilon_0} D_m \sin(\omega t + \beta z)\hat{a}_z$$

From the relations, we determine: $\mathbf{B} = \dfrac{D_m \sin(\omega t + \beta z)}{\epsilon_0}\hat{a}_z$

$$\mathbf{J_d} = \frac{\partial \mathbf{D}}{\partial t}$$

Taking the derivative of $\mathbf{D}$: $\mathbf{J_d} = \dfrac{\partial}{\partial t}\left(D_m \sin(\omega t + \beta z)\hat{a}_x\right)$

$$\mathbf{J_d} = D_m \omega \cos(\omega t + \beta z)\hat{a}_x$$

Thus, the **displacement current density** is: $\mathbf{J_d} = D_m \omega \cos(\omega t + \beta z)\hat{a}_x$

9) Calculate **D**, **J$_D$**. In free space **E**=cos(ωt-50x)a$_y$ V/m

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{D} = \varepsilon_0 \cos(\omega t - 50x)\,\hat{a}_y$$

$$\boxed{\vec{D} = 8.854 \times 10^{-12} \cos(\omega t - 50x)\,\hat{a}_y\,\mathrm{C/m^2}}$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} = \cos(\omega t - 50x) \Rightarrow \frac{\partial \vec{E}}{\partial t} = -\omega \sin(\omega t - 50x)$$

$$\vec{J}_d = -\varepsilon_0 \omega \sin(\omega t - 50x)\,\hat{a}_y$$

$$\boxed{\vec{J}_d = -8.854 \times 10^{-12}\,\omega\,\sin(\omega t - 50x)\,\hat{a}_y\,\mathrm{A/m^2}}$$

10) Find the electric flux density and volume charge density if The electric field E=X^{2}a$_x$Whose in a medium ε_r=2.

- Electric field: $\vec{E} = x^2\,\hat{a}_x \quad \mathrm{V/m}$
- Medium with relative permittivity: $\varepsilon_r = 2$
- Free space permittivity: $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$
- So: $\varepsilon = \varepsilon_r \cdot \varepsilon_0 = 2\times 8.854 \times 10^{-12} = 1.7708 \times 10^{-11}\,\mathrm{F/m}$

$$\vec{D} = \varepsilon\vec{E} = \left(1.7708 \times 10^{-11}\right) x^2\,\hat{a}_x \quad \mathrm{C/m^2}$$

$$\boxed{\vec{D} = 1.7708 \times 10^{-11} x^2\,\hat{a}_x \quad \mathrm{C/m^2}}$$

From **Gauss's Law** (differential form):

$$\rho_v = \nabla \cdot \vec{D}$$

Since $\vec{D} = D_x(x)\,\hat{a}_x$, only partial with respect to x matters:

$$\rho_v = \frac{\partial D_x}{\partial x} = \frac{\partial}{\partial x}\left(1.7708 \times 10^{-11} x^2\right) = 2\times 1.7708 \times 10^{-11} x = 3.5416 \times 10^{-11} x$$

$$\boxed{\rho_v = 3.5416 \times 10^{-11} x \quad \mathrm{C/m^3}}$$

11) Find **H** at (0, 0, -1) cm of a thin ring of radius 5 cm is placed on plane z =1 cm so that its center is at (0, 0, 1) cm. If the ring carries 50 mA along a_ϕ

A thin ring of radius 5 cm is placed on plane z = 1 cm so that its center is at (0, 0, 1) cm. If the ring carries 50 mA along $\bar{a}_\phi$. Find $\bar{H}$ at

(a) (0, 0, –1) cm

Solution

(a) Here ρ = 5 cm, h = 2 cm and I = 50 mA

We have $\bar{H} = \dfrac{I\rho^2\,\bar{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$

$$\bar{H} = \frac{50\times 10^{-3}\,(5\times 10^{-2})^2\,\bar{a}_z}{2\left[(5\times 10^{-2})^2 + (2\times 10^{-2})^2\right]^{3/2}}$$

$$\bar{H} = \frac{125\,\bar{a}_z}{2[29]^{3/2}}$$

$$\bar{H} = 400\bar{a}_z \quad \mathrm{mA/m}$$

12) Find the current density **J** in the both the regions (r<a and r>a) of a long straight conductor with radius 'a' has a magnetic field strength H = $\frac{Ir}{2\pi a^2}\boldsymbol{a_\emptyset}$ within the conductor (r<a) and H = $\frac{I}{2\pi r}\boldsymbol{a_\emptyset}$ outside the conductor (r>a).

In cylindrical coordinates, for a purely ϕ-directed field $\mathbf{H} = H_\phi(r)\,\mathbf{a}_\phi$, the curl is:

we have $\bar{J} = \nabla \times \bar{H}$

Given $\bar{H} = \frac{Ir}{2\pi a^2}\bar{a}_\phi$ within the conductor (r < a)

which has cylindrical symmetry, here ρ = r,

$$\bar{J} = \nabla \times \bar{H} = \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_r & rH_\phi & H_z \end{vmatrix}$$

$$= \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & r\frac{Ir}{2\pi a^2} & 0 \end{vmatrix} = \frac{I}{\pi a^2}\bar{a}_z \ \ \mathrm{A/m^2}$$

And also given $\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi$ outside the conductor (r > a)

$$\bar{J} = \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & r\frac{I}{2\pi r} & 0 \end{vmatrix} = 0 \ \ \mathrm{A/m^2}$$

13) Determine the magnetic flux for the surface described by (i) ρ =1 m., $0 \le \emptyset \le \frac{\pi}{2}$ (ii) a sphere of radius 2 m. If the magnetic field is of the form H = $\frac{1}{\rho}cos\emptyset\, \boldsymbol{a_\rho}$ A/m

Solution:

We have magnetic flux $\psi = \int_s \bar{B}.d\bar{s} = \mu_0 \int_s \bar{H}.d\bar{s}$

(a) here $d\bar{s} = \rho d\phi dz \bar{a}_\rho$ (cylindrical symmetry)

$$\psi = \mu_0 \int_s \frac{1}{\rho} \cos\phi \bar{a}_\rho . \rho d\phi dz \bar{a}_\rho = \mu_0 \int_{z=0}^{2} dz \int_{\phi=0}^{\pi/2} \cos\phi d\phi = 2\mu_0 = 25.13\times10^{-7} \text{ Wb}$$

(b) here $d\bar{s} = r^2 \sin\theta d\theta d\phi \bar{a}_r$ (spherical symmetry)

$$\psi = \mu_0 \int_s \frac{1}{r} \cos\phi \bar{a}_r . r^2 \sin\theta d\theta d\phi \bar{a}_r = \mu_0 r \int_{\theta=0}^{\pi} \sin\theta d\theta \int_{\phi=0}^{2\pi} \cos\phi d\phi = 0 \text{ Wb}$$

14) Find the flux density at the center of a square loop of 10 turns carrying a current of 10 A. The loop is in air and has a side of 2 m.

Solution

We have $\bar{H} = \frac{I\sqrt{2}}{\pi a} \bar{a}_z$

$\therefore$ magnetic flux density is $\bar{B} = \frac{\mu_0 I \sqrt{2}}{\pi a} \bar{a}_z$

Here no. of turns N = 10

Total current I = N × current in each turn

I = 10 × 10 = 100 A

and a = 1 m

$$\therefore \qquad \bar{B} = \frac{\mu_0 100\sqrt{2}}{\pi \times 1} \bar{a}_z$$

$$\bar{B} = \frac{4\pi \times 10^{-7} \times 100\sqrt{2}}{\pi \times 1} \bar{a}_z = 56.569 \mu \bar{a}_z \text{Tesla}$$

15) Calculate **B** at (-1, 2, 5) of a current distribution gives rise to the vector magnetic potential **A** = $x^2 y \boldsymbol{a_x} + y^2 x \boldsymbol{a_y} - 4xyz \boldsymbol{a_z}$ Wb/m

$$\text{(a)} \quad \bar{B} = \nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & y^2 x & -4xyz \end{vmatrix}$$

$$\bar{B} = \bar{a}_x(-4xz - 0) - \bar{a}_y(-4yz - 0) + \bar{a}_z(y^2 - x^2)$$

$$\bar{B} = -4xz\bar{a}_x + 4yz\,\bar{a}_y + (y^2 - x^2)\bar{a}_z$$

$$\bar{B}_{(-1,\,2,\,3)} = 20\bar{a}_x + 40\bar{a}_y + 3\bar{a}_z \text{ Wb/m}^2$$

UNIT-3

1) Determine the propagation constant and intrinsic impedance of the medium, if $\varepsilon_r = 9$, $\mu = \mu_0$ for the medium in which a wave with a frequency f=0.3 GHz is propagating when σ=0.

Solution The expression for propagation constant, γ is

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu\epsilon} \text{ for } \sigma = 0$$

As $\mu_r = 1, \epsilon_r = 9, \sigma = 0, f = 0.3 \text{ GHz}$

$$\gamma = j\,2\pi\times3\times10^8\times\sqrt{4\pi\times10^{-7}\times9\times8.854\times10^{-12}}$$

$$= j\,6\pi\times10^8\times2\times3\sqrt{8.854\times10^{-19}\times\pi}$$

$$= j\,36\pi\times10^8\times10^{-9}\times1.66780$$

$$\boxed{\gamma = j\,18.8624\,(1/\text{m})}$$

Intrinsic impedance,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \sqrt{\frac{\mu_0}{9\epsilon_0}} = \frac{1}{3}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{3}\times120\pi$$

$$\boxed{\eta = 40\pi\Omega}$$

2) Calculate the frequency and permittivity of the medium when The wavelength of an x-directed plane wave in a lossless medium is 0.25 m and the velocity of propagation is 1.5 X 10^{10} cm/s. The wave has z-directed electric field with an amplitude equal to 10 V/m. (medium has $\mu = \mu_0$).

Solution $\upsilon = 1.5\times10^{10}\,\text{cm/sec} = 1.5\times10^8\,\text{m/s}$

Frequency of the wave,

$$f = \frac{\upsilon}{\lambda} = \frac{1.5\times10^8}{25\times10^{-2}} = 0.06\times10^{10}$$

$$\boxed{f = 600\text{ MHz}}$$

We have

$$\upsilon = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

$$= \frac{\upsilon_0}{\sqrt{\epsilon_r}} = 1.5\times10^8$$

$$\sqrt{\epsilon_r} = \frac{3\times10^8}{1.5\times10^8} = 2$$

$$\boxed{\epsilon_r = 4}$$

3) Calculate the Phase constant, Phase Velocity and Equation for H_Z, if $E_Z = H_Y=0$, for a uniform plane wave travelling in a X-Direction in the free space $E_Y=10Sin\ (2\pi10^8t-\beta x)$.

Given $E_y=10\sin(2\pi\ 10^8\ t - \beta x)$

$$\omega = 2\pi\ 10^8;\quad \beta^2 = \omega^2\mu_0\epsilon_0 = (2\pi10^8)^2 \times 8.854\times10^{-12}\times 4\pi\times10^{-7}$$

The Phase constant $\beta = 2.096$

The Phase Velocity $\quad v = \frac{\omega}{\beta} = \frac{2\pi\times10^8}{2.096} = 2.99\times10^8\ \text{m/s}$

We have $\quad \frac{E_y}{H_z} = 377$

$$\Rightarrow \qquad H_z = \frac{10}{377}\sin(2\pi10^8 t - \beta x)$$

4) calculate the following the average power if the magnetic field intensity (**H)** has a magnitude of 5 mA/m in a medium defined by$\epsilon_r = 4, \mu_r = 1$.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \quad (\text{since } \mu_r = 1)$$

Where $\eta_0 = 377\,\Omega$ (intrinsic impedance of free space):

$$\eta = \frac{377}{\sqrt{4}} = \frac{377}{2} = 188.5\,\Omega$$

$$E = \eta H = 188.5 \times 5 \times 10^{-3} = 0.9425 \text{ V/m}$$

$$\langle S \rangle = \frac{1}{2} E H$$

$$\langle S \rangle = \frac{1}{2} \times 0.9425 \times 5 \times 10^{-3} = 2.356 \times 10^{-3} \text{ W/m}^2$$

5) Calculate the i) Velocity of Propagation ii) Intrinsic impedance in free space an in Material iii) Wavelength in free space in material, when f=1GHz, an EM Wave Propagated through a material μ_r =5 and $\varepsilon_r = 10$.

Solution

(a) $v = \dfrac{1}{\sqrt{\mu_0 \mu_r \in_0 \in_r}} = 42.4 \text{ M m/s.}$

(b) Intrinsic impedance

in free space $\eta = \sqrt{\dfrac{\mu_0}{\in_0}} = \sqrt{\dfrac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 377\ \Omega$

in material $\eta = \sqrt{\dfrac{\mu_0 \mu_r}{\in_0 \in_r}} = \sqrt{\dfrac{4\pi \times 10^{-7} \times 5}{8.854 \times 10^{-12} \times 10}} = 266.39\ \Omega$

(c) Wavelength

in free space $\lambda = \dfrac{v_0}{f} = \dfrac{3 \times 10^8}{1 \times 10^8} = 3 \text{ m}$

in material $\lambda = \dfrac{v}{f} = \dfrac{42.4 \times 10^6}{1 \times 10^8} = 0.424 \text{ m}$

6) Determine the (i) magnitude of the electric field for the plane wave in free space (ii) magnitude of the electric field when the wave propagates in a medium which is characterised by $\sigma = 0$, $\mu = \mu_0$ and $\epsilon = 4\varepsilon_0$ When the amplitude of the magnetic field in a plane wave is 2 A/m

Solution:

Use wave impedance for free space

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$$

Electric field:

$$E = \eta_0 \cdot H = 120\pi \cdot 2 = 240\pi \text{ V/m}$$

$\mu = \mu_0$ and $\epsilon = 4\varepsilon_0$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{2} \cdot 120\pi = 60\pi\ \Omega$$

$$E = \eta \cdot H = 60\pi \cdot 2 = 120\pi \text{ V/m}$$

$$\boxed{E = 120\pi \text{ V/m}}$$

7) Calculate the Depth of penetration, of an EM wave in copper at f = 60 Hz and f = 100 MHz for copper σ = 5.8 X 10^7 mho/m $\epsilon_r = 1, \mu_r = 1$

Solution:

Conductivity, $\sigma = 5.8 \times 10^7\,\mathrm{S/m}$

Relative permeability, $\mu_r = 1$

Relative permittivity, $\varepsilon_r = 1$ *(not needed in formula)*

Use $\mu = \mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$

Skin Depth:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{where} \quad \omega = 2\pi f$$

$f = 60\,\mathrm{Hz}$:

$$\omega = 2\pi \times 60 = 377\,\mathrm{rad/s}$$

$$\delta = \sqrt{\frac{2}{377 \cdot 4\pi \times 10^{-7} \cdot 5.8 \times 10^7}} \approx \boxed{8.53\,\mathrm{mm}}$$

$f = 100\,\mathrm{MHz} = 1 \times 10^8\,\mathrm{Hz}$:

$$\omega = 2\pi \times 10^8 = 6.28 \times 10^8\,\mathrm{rad/s}$$

$$\delta = \sqrt{\frac{2}{6.28 \times 10^8 \cdot 4\pi \times 10^{-7} \cdot 5.8 \times 10^7}} \approx \boxed{6.61\,\mu\mathrm{m}}$$

8) Determine the angle by which **E** tilts as the wave crosses the boundary when an EM wave travelling in free space incidents on a dielectric medium with relative constant is 2 at an angle 45^0.

Solution:

$$\epsilon_1 = \epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{Wb/m}$$

$$\epsilon_2 = \epsilon_0\ \epsilon_r = 2 \times 8.854 \times 10^{-12}\ \mathrm{Wb/m}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin\theta_2 = \sin\theta_1\sqrt{\frac{\epsilon_1}{\epsilon_2}} = 0.5$$

$$\theta_2 = 30^{\circ}$$

9) (i) Determine the Phase Velocity of the wave (ii) Amplitude and Direction of magnetic field intensity If the electric field E has only a 'y' component with amplitude of 20 V/m. A plane wave travelling in +ve X-direction in a loss less unbounded medium having permeability 4.5 times that of free space and a permittivity twice that of free space.

Solution

(a) Phase velocity $v = \dfrac{1}{\sqrt{\mu \in}} = \dfrac{1}{\sqrt{4.5\mu_0 \, 2 \in_0}}$

$= 1 \times 10^8$ m/s

(b) $H_z = \sqrt{\dfrac{\in}{\mu}} E_y = 0.0354$ A / m, it's direction is along 'z'.

10)

Determine Maximum Energy Density in a medium defined by $\epsilon_r = 4, \mu_r = 1$, when given an electric field **E** is 942.5mV/meter, and magnetic field **H** is 5 mA/meter.

Solution:

$$W = W_E + W_H = \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2$$

$$\varepsilon = \varepsilon_r \varepsilon_0 = 4 \cdot 8.854 \times 10^{-12} = 35.416 \times 10^{-12}\,\mathrm{F/m}$$

$$\mu = \mu_r \mu_0 = 1 \cdot 4\pi \times 10^{-7} = 1.2566 \times 10^{-6}\,\mathrm{H/m}$$

Electric energy density:

$$W_E = \frac{1}{2} \cdot 35.416 \times 10^{-12} \cdot (942.5 \times 10^{-3})^2 = \frac{1}{2} \cdot 35.416 \times 10^{-12} \cdot 0.8883 \approx 15.73 \times 10^{-12}\,\mathrm{J/m^3}$$

Magnetic energy density:

$$W_H = \frac{1}{2} \cdot 1.2566 \times 10^{-6} \cdot (5 \times 10^{-3})^2 = \frac{1}{2} \cdot 1.2566 \times 10^{-6} \cdot 25 \times 10^{-6} \approx 15.71 \times 10^{-12}\,\mathrm{J/m^3}$$

$$W = W_E + W_H = 15.73 \times 10^{-12} + 15.71 \times 10^{-12} = \boxed{31.44 \times 10^{-12}\,\mathrm{J/m^3}}$$

$$\boxed{W = 31.44\,\mathrm{pJ/m^3}}$$

11) Determine the average power density absorbed by copper if a plane wave with a frequency of 2 MHz is incident upon a copper conductor normally. The wave has an electric field amplitude of **E** = 2 mV/m. Copper has $\epsilon_r = 1, \mu_r = 1$ and σ= 5.8 X 10^7 Ω/m

Solution:

Angular frequency: $\omega = 2\pi f$

Skin depth for a good conductor: $\delta = \sqrt{\dfrac{2}{\omega\mu\sigma}}$

Average power absorbed: $P_{\text{avg}} = \dfrac{1}{2} \cdot \dfrac{E^2}{\delta \cdot \sigma}$

Angular frequency: $\omega = 2\pi \cdot 2 \times 10^6 = 4\pi \times 10^6 \approx 1.2566 \times 10^7\,\mathrm{rad/s}$

Skin depth: $$\delta = \sqrt{\frac{2}{(1.2566 \times 10^7)(4\pi \times 10^{-7})(5.8 \times 10^7)}} \approx 46.73 \times 10^{-6}\,\mathrm{m} = 46.73\,\mu\mathrm{m}$$

Power density: $$P_{\text{avg}} = \frac{1}{2} \cdot \frac{(2 \times 10^{-3})^2}{46.73 \times 10^{-6} \cdot 5.8 \times 10^7}$$

$$P_{\text{avg}} \approx 7.38 \times 10^{-10}\,\mathrm{W/m^2}$$

12) Determine (i) The Intrinsic Impedance, (ii) the Propagation Constant, (iii) the Phase Velocity for a Dry ground has a conductivity of 5 X 10^{-4} mhos/m and relative dielectric constant of 10 at a frequency of 500 MHz.

Solution

Given $\sigma = 5 \times 10^{-4}$ mhos / m, $\varepsilon_r = 10$ and $f = 500$ MHz

Attenuation constant $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = 2513.27$

Phase shift constant $\beta = \alpha = 2513.27$

Propagation constant $\gamma = \alpha + j\beta = 2513.27 + j\,2513.27$

$= 3554.3 \angle 45^\circ$

Phase velocity $V = \frac{\omega}{\beta} = 40$ Mm / s

intrinsic impedance $\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = 35.543\sqrt{j}$

$= 35.543\angle 45^\circ$

13) Determine the displacement current density and conduction current density if the same field exists in a medium whose conductivity is given by 2.0X10^3mho/cm and electric field strength of a radio broadcast signal at a TV receiver is given by E = 5.0cos(wt - by)$\mathbf{a_z}$, V/m.

Solution:

Electric Field: $\vec{E} = 5.0\cos(\omega t - \beta y)\hat{a}_z\ \mathrm{V/m}$

Conductivity: $\sigma = 2.0 \times 10^3\ \mathrm{mho/cm} = 2.0 \times 10^5\ \mathrm{S/m}$

Permittivity of free space: $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$

$$\vec{J}_c = \sigma \vec{E}$$

$$\vec{J}_c = 2.0 \times 10^5 \cdot 5\cos(\omega t - \beta y)\hat{a}_z$$

$$\boxed{\vec{J}_c = 1.0 \times 10^6 \cos(\omega t - \beta y)\hat{a}_z\ \mathrm{A/m^2}}$$

$$\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial \vec{E}}{\partial t} = -5\omega \sin(\omega t - \beta y)\hat{a}_z$$

$$\vec{J}_d = 8.854 \times 10^{-12} \cdot (-5\omega \sin(\omega t - \beta y))\hat{a}_z$$

$$\boxed{\vec{J}_d = -4.427 \times 10^{-11} \cdot \omega \cdot \sin(\omega t - \beta y)\hat{a}_z \text{ A/m}^2}$$

14) Determine the Propagation Constant and velocity of EM Wave at 500 KHz for a medium in which σ=0, μ_r =1 and $\varepsilon_r = 15$.

Solution:

Solution:

$$\frac{\sigma}{\omega \in} = 0 = \text{Perfect dielectric}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu \in} = 0.041$$

$$\gamma = \alpha + j\beta = 0.041 \angle 90^{\circ}$$

Wave travels with velocity $v = \dfrac{\omega}{\beta} = 76.624$ Mm/s

15) Determine **B, H, D** for an electric field in a medium which is source-free is given by **E** = 1.5cos(10^8t – βz)$\mathbf{a}_x$ V/m, where E_m is the amplitude, ω is the angular frequency and β is the phase constant.

Solution:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0 = 377\,\Omega \quad (\text{since } \mu_r = \varepsilon_r = 1)$$

$$\mathbf{H} = \frac{E_m}{\eta} \cos(\omega t - \beta z)\,\mathbf{a}_y$$

$$\boxed{\mathbf{H} = \frac{1.5}{377} \cos(10^8 t - \beta z)\,\mathbf{a}_y \approx 3.98 \times 10^{-3} \cos(10^8 t - \beta z)\,\mathbf{a}_y \quad (\text{A/m})}$$

$$\mathbf{B} = \mu\mathbf{H} \quad \text{and} \quad \mu = \mu_r\mu_0 = 1 \cdot 4\pi \times 10^{-7}\,\text{H/m}$$

$$\mathbf{B} = 4\pi \times 10^{-7} \cdot 3.98 \times 10^{-3} \cos(10^8 t - \beta z)\,\mathbf{a}_y \approx 5.0 \times 10^{-9} \cos(10^8 t - \beta z)\,\mathbf{a}_y \quad (\text{T})$$

$$\mathbf{D} = \varepsilon\mathbf{E} \quad \text{and} \quad \varepsilon = \varepsilon_r\varepsilon_0 = 1 \cdot 8.854 \times 10^{-12}\,\text{F/m}$$

$$\mathbf{D} = 8.854 \times 10^{-12} \cdot 1.5 \cos(10^8 t - \beta z)\,\mathbf{a}_x \approx 1.328 \times 10^{-11} \cos(10^8 t - \beta z)\,\mathbf{a}_x \quad (\text{C/m}^2)$$

UNIT-4

1) Find the inductance and capacitance of the line if a transmission line with air as dielectric has $Z_0 = 50\Omega$ and a phase constant of 3.0 rad/m at 10MHz.

Solution:

A line with air dielectric is lossless as $\sigma = 0$.

$$R = 0 = G \text{ and } \alpha = 0$$

$$z_0 = R_0 = 50\Omega = \sqrt{\frac{L}{C}}$$

$$\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC}$$

$$\frac{R_0}{\beta} = \frac{\sqrt{L/C}}{2\pi f\sqrt{LC}} = \frac{1}{\omega C}$$

$$C = \frac{\beta}{\omega R_0} = \frac{3}{2\pi \times 10^7 \times 50} = 0.9549\times10^{-9}$$

$$\boxed{C = 954.9 \text{ pF/m}}$$

$$\sqrt{\frac{L}{C}} = R_0 = 50\Omega$$

$$L = 50^2 C = 2500\times 954.9\times10^{-12}$$

$$\boxed{L = 2387.25 \text{ nH/m}}$$

2) Find R, L, G, C and wavelength at 0.1 GHz. If a transmission line in which no distortion is present has the following parameters Z_0=50 Ω $\alpha = 0.02\text{m}^{-1}$, $v=0.6v_0$

Solution:

For a distortionless line the condition is

$$RC = GL \text{ or } G = \frac{RC}{L}$$

and

$$z_0 = \sqrt{\frac{L}{C}}$$

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}}$$

$$= \frac{R}{z_0}$$

and hence

$$R = \alpha z_0$$

$$= 20\times 10^{-3}\times 50 = 1.0\,\Omega/\text{m}$$

$$L = \frac{z_0}{\upsilon} = \frac{50}{0.6\times 3\times10^8} = 277 \text{ nH/m}$$

$$G = \frac{\alpha^2}{R} = \frac{400\times10^{-6}}{1.0}$$

$$G = 400\mu \text{ mho/m}$$

$$C = \frac{1}{\upsilon z_0} = \frac{1}{0.6\times 3\times 10^8\times50} = 111.1 \text{ pF/m}$$

$$\lambda = \frac{\upsilon}{f} = \frac{0.6\times 3\times10^8}{0.1\times10^9} = 1.8 \text{ m}$$

3) Find the characteristic impedance of the line when a copper coaxial line has an outside tubing of thickness 1.8 mm and its outside diameter is 30mm. The thickness of the inner tubing is 1.0mm and its outside diameter is 8 mm.

Solution:

Solution Diameter of the outside conductor is

$$d_1 = 30 - 2\times1.8 = 26.4 \text{ mm}$$

Diameter of the inner conductor is

$$d_2 = 8 - 2\times1.0 = 6.0 \text{ mm}$$

For a coaxial cable, z_0 is

$$z_0 = 138 \log\left(\frac{d_1}{d_2}\right)$$

$$= 138 \log\left(\frac{26.4}{6}\right)$$

$$= 138 \log_{10} 4.40$$

$$\boxed{z_0 = 88.79\Omega}$$

4) Find the characteristic impedance for sections of a line 10 metre long and 500 metre long, if lossless transmission line used in a TV receiver has a capacitance of 50pF /m and an inductance of 200 nH/m.

Solution:

$$z_0 = \sqrt{\frac{L}{C}}$$

The inductance, L of the line

$$= 200 \text{ nH/m}$$

For 10 m line,

$$L = 200\times 10^{-9} \times 10 = 2000 \times 10^{-9}$$

$$= 2 \times 10^{-6} \text{ H}$$

$$C = 50 \text{ pF/m}$$

For 10 m line,

$$C = 50\times 10^{-12} \times 10 = 5 \times 10^{-10} \text{ F}$$

The characteristic impedance, z_0

$$z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2\times 10^{-6}}{5\times 10^{-10}}}$$

$$= \sqrt{0.4 \times 10^4}$$

$$\boxed{z_0 = 63.245\,\Omega}$$

The inductance of 500 m line

$$L = 200\times 10^{-9} \times 500$$

$$= 10{,}0000 \times 10^{-9}$$

$$L = 10^{-4} \text{ H}$$

The capacitance of 500 m line

$$C = 50\times 10^{-12} \times 500$$

$$= 25000 \times 10^{-12} = 25 \times 10^{-9} \text{ F}$$

The characteristic impedance, z_0 of 500 m line

$$z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{10^{-4}}{25\times 10^{-9}}}$$

$$= \sqrt{0.04\times 10^{-4} \times 10^{9}}$$

$$= \sqrt{0.4\times 10^{-5} \times 10^{9}}$$

$$= \sqrt{0.4 \times 10^{4}}$$

$$= 0.632 \times 10^{2}$$

$$\boxed{z_0 = 63.2\,\Omega}$$

5) (i) Find the time delay for a cable 1 m long, (ii) propagation velocity, and (iii) propagation delay over a cable length of 10 m. If a signal of 30 MHz is transmitted through a coaxial cable which has a capacitance of 30 pF/m and an inductance of 500 nH/m.

Solution:

The time delay for 1 m long cable is

$$t_d = \sqrt{LC}$$

$$= \sqrt{(500\times10^{-9})\,(30\times10^{-12})}$$

$$= \sqrt{15\times10^{-18}}$$

$$\boxed{t_d = 3.87 \text{ n sec}}$$

Velocity of propagation,

$$\upsilon_p = \frac{1 \text{ metre}}{3.87\times10^{-9}}$$

$$\boxed{\upsilon_p = 2.5839\times10^{8} \text{ m/s}}$$

The inductance of 10 metre cable is

$$L = 500\times 10^{-9}\times10 = 5\times10^{-6} \text{ H}$$

The capacitance of 10 metre cable is

$$C = 30\times 10^{-12}\times10 = 3\times10^{-10} \text{ F}$$

The time delay,

$$t_d = \sqrt{LC} = \sqrt{5\times 10^{-6}\times 3\times10^{-10}}$$

$$= 3.87\times10^{-8}$$

$$\boxed{t_d = 38.7 \text{ ns}}$$

6) Find the Characteristics Impedance and Propagation Constant of the line when A high frequency line has the following primary constants L=1.2mH/Km, C=0.05μF/Km. R = G = negligible.

Solution:

- Inductance per unit length: $L = 1.2 \times 10^{-3}\,\mathrm{H/km}$
- Capacitance per unit length: $C = 0.05 \times 10^{-6}\,\mathrm{F/km}$
- Frequency: $f = 10\,\mathrm{MHz} = 10 \times 10^{6}\,\mathrm{Hz}$
- $R = 0$, $G = 0$ (lossless line)

Characteristics Impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.2 \times 10^{-3}}{0.05 \times 10^{-6}}}$$

$$= \sqrt{24 \times 10^3} = \sqrt{24000} \approx 154.92\,\Omega$$

Propagation Constant:

$$\gamma = j\omega\sqrt{LC}$$

$$\omega = 2\pi f = 2\pi \times 10^7\,\mathrm{rad/s}$$

$$LC = (1.2 \times 10^{-3})(0.05 \times 10^{-6}) = 6 \times 10^{-11}$$

$$\sqrt{LC} = \sqrt{6 \times 10^{-11}} \approx 7.745 \times 10^{-6}$$

$$\gamma = j(2\pi \times 10^7)(7.745 \times 10^{-6}) \approx j(48.63)\ \text{per km}$$

7) Find the Phase Constant and the Reflection coefficient of a line of length 50 m. Characteristic Impedance Z_0 is 50Ω and wavelength on the line is 0.45 m of lossless transmission line is terminated in a load impedance of 30-j23Ω.

Solution:

$$z_L = (30 - j23)\,\Omega$$

$$z_0 = 50\,\Omega$$

$$\lambda = 0.45\,\mathrm{m}$$

$$l = 50\,\mathrm{m}$$

$$\text{Phase constant,} \quad \beta = \frac{2\pi}{\lambda} = 13.9626\ \mathrm{rad/m}$$

$$\text{Reflection coefficient,} \quad \rho = \frac{z_L - z_0}{z_L + z_0}$$

$$= \frac{30 - j23 - 50}{30 - j23 + 50}$$

$$= \frac{-(20 + j23)}{(80 - j23)}$$

$$\boxed{\rho = -(0.1431 + j0.3071)}$$

8) Find the Propagation Constant and Intrinsic Impedance of the medium when σ =0. If $\varepsilon_r = 9$, $\mu = \mu_0$ for the medium in which a wave with a frequency of f = 0.3 GHz is propagating.

Solution:

- $\sigma = 0$ (perfect dielectric)
- $\varepsilon_r = 9$
- $\mu = \mu_0$
- Frequency $f = 0.3\,\text{GHz} = 0.3 \times 10^9\,\text{Hz}$
- Propagation constant: $\gamma = j\omega\sqrt{\mu\varepsilon}$
- Intrinsic impedance: $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

Calculate $\omega = 2\pi f = 2\pi \times 0.3 \times 10^9 = 1.884 \times 10^9\,\text{rad/s}$

- $\mu = \mu_0 = 4\pi \times 10^{-7}\,\text{H/m}$
- $\varepsilon = \varepsilon_r\varepsilon_0 = 9 \times 8.854 \times 10^{-12} = 7.9686 \times 10^{-11}\,\text{F/m}$

Propagation constant $\gamma = j\omega\sqrt{\mu\varepsilon}$

$$\mu\varepsilon = (4\pi \times 10^{-7}) \times (7.9686 \times 10^{-11}) \approx 1.0008 \times 10^{-16}$$

$$\sqrt{\mu\varepsilon} \approx \sqrt{1.0008 \times 10^{-16}} \approx 1.0004 \times 10^{-8}$$

$$\gamma = j \times 1.884 \times 10^9 \times 1.0004 \times 10^{-8}$$

$$\gamma \approx j \times 18.85\,(1/\text{m})$$

Intrinsic Impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\,\Omega$$

$$\eta = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

$$\eta = \frac{120\pi}{3} = 40\pi\,\Omega$$

9) Find out i) Propagation velocity ii) Phase constant at an operating frequency of 100 KHz iii. Characteristic impedance of the line when a lossless transmission line of length 100m has an inductance of 28 μH and capacitance of 20nF.

Solution:

Given:

- Length of line, $l = 100\,\text{m}$
- Inductance of line, $L = 28\,\mu\text{H} = 28 \times 10^{-6}\,\text{H}$
- Capacitance of line, $C = 20\,\text{nF} = 20 \times 10^{-9}\,\text{F}$
- Frequency, $f = 100\,\text{kHz} = 10^5\,\text{Hz}$

Per unit length:

- $L' = \frac{28\times10^{-6}}{100} = 0.28 \times 10^{-6}\,\mathrm{H/m}$
- $C' = \frac{20\times10^{-9}}{100} = 0.20 \times 10^{-9}\,\mathrm{F/m}$

(i) Propagation velocity $v_p = \dfrac{1}{\sqrt{L'C'}}$

Substitute values: $$v_p = \frac{1}{\sqrt{0.28 \times 10^{-6} \times 0.20 \times 10^{-9}}}$$

$$= \frac{1}{\sqrt{5.6 \times 10^{-17}}}$$

$$= \frac{1}{7.48 \times 10^{-9}}$$

$$v_p = 1.336 \times 10^{8}\,\mathrm{m/s}$$

(ii) Phase constant $\beta = \dfrac{\omega}{v_p}$ where $\omega = 2\pi f$

$$\omega = 2\pi \times 10^5 = 6.283 \times 10^5\,\mathrm{rad/s}$$

$$\beta = \frac{6.283 \times 10^5}{1.336 \times 10^8}$$

$$\beta = 4.702 \times 10^{-3}\,\mathrm{rad/m}$$

(iii) Characteristic impedance $Z_0 = \sqrt{\dfrac{L'}{C'}}$

$$Z_0 = \sqrt{\frac{0.28 \times 10^{-6}}{0.20 \times 10^{-9}}}$$

$$= \sqrt{1400}$$

$$Z_0 = 37.42\,\Omega$$

10) Find out the characteristic impedance of the line when a two-wire open air line, whose diameter is 2.588 mm, is used in several applications and the wires are spaced at 290 mm between the centres.

Solution Radius of the wire

$$r = \frac{d}{2} = \frac{2.588}{2} = 1.294\ \mathrm{mm}$$

Spacing between the wires is $s = 290$ mm

The characteristic impedance of the two-wire open air line is

$$z_0 = 276 \log_{10}\frac{s}{r} = 276 \log_{10}\frac{290}{1.294} = 276 \log_{10} 224.11$$
$$= 276 \times 2.350$$

$$\boxed{z_0 = 648.7\ \Omega}$$

11) Find the Capacitance C per unit length A lossless transmission line has a characteristic impedance of ZO = 75 Ω and inductance L = 300 nH/m.
Solution:

$$Z_0 = \sqrt{\frac{L}{C}} \quad \Rightarrow \quad C = \frac{L}{Z_0^2}$$

$$C = \frac{300 \times 10^{-9}}{(75)^2} = \frac{300 \times 10^{-9}}{5625} = 53.33 \text{ pF/m}$$

$$C = 53.33 \text{ pF/m}$$

12) Find the phase constant β of given a transmission line has a propagation velocity $V_P = 2 \text{ X } 10^8$ m/s and operate at frequency f = 100 MHz.
Solution:

Given the relation: $v_p = \frac{\omega}{\beta} \quad \Rightarrow \quad \beta = \frac{\omega}{v_p}$

Since $\omega = 2\pi f$, we substitute: $\beta = \frac{2\pi f}{v_p}$

Substituting the given values:

- $f = 100 \times 10^6 \text{ Hz}$
- $v_p = 2 \times 10^8 \text{ m/s}$

Thus: $\beta = \frac{2\pi \times 100 \times 10^6}{2 \times 10^8} = \pi \text{ rad/m}$

$$\boxed{\beta = \pi \text{ rad/m}}$$

13) Find the normalised impedance magnitude for a transmission line which is terminated in a normalised Impedance Z_n, VSWR = 2.
Solution:

Normalised impedance, z_n

$$z_n = \frac{z_L}{z_0}$$

$$S = \text{VSWR} = 2 = \frac{1 + |\rho|}{1 - |\rho|}$$

$$|\rho| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$|\rho| = \left|\frac{z_L - z_0}{z_L + z_0}\right|$$

$$= \left|\frac{\frac{z_L}{z_0} - 1}{\frac{z_L}{z_0} + 1}\right| = \frac{1}{3}$$

$$\boxed{|z_n| = 2}$$

14) Find the inductance per unit length L and capacitance per unit length C of a lossless transmission line has characteristic impedance Z_0 is 60Ω and a propagation velocity V_P is 2.5 X 10^8 m/s.

Solution:

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{and} \quad v_p = \frac{1}{\sqrt{LC}}$$

$$LC = \frac{1}{v_p^2}$$

$v_p = 2.5 \times 10^8\ \mathrm{m/s}$:

$$LC = \frac{1}{(2.5 \times 10^8)^2} = \frac{1}{6.25 \times 10^{16}} = 1.6 \times 10^{-17}\ \mathrm{H \cdot F/m^2}$$

$$Z_0^2 = \frac{L}{C} \quad \Rightarrow \quad L = Z_0^2 \times C$$

Substituting $L = Z_0^2 \times C$ into $LC = 1.6 \times 10^{-17}$:

$$(Z_0^2)C^2 = 1.6 \times 10^{-17} \quad \Rightarrow \quad (60)^2 C^2 = 1.6 \times 10^{-17}$$

$$\Rightarrow \quad 3600C^2 = 1.6 \times 10^{-17} \quad \Rightarrow \quad C^2 = \frac{1.6 \times 10^{-17}}{3600} = 4.444 \times 10^{-21}$$

Taking square root: $C = \sqrt{4.444 \times 10^{-21}} = 2.107 \times 10^{-10}\ \mathrm{F/m} = 210.7\ \mathrm{pF/m}$

Now using: $L = Z_0^2 \times C = 3600 \times 2.107 \times 10^{-10} = 7.585 \times 10^{-7}\ \mathrm{H/m} = 758.5\ \mathrm{nH/m}$

15) Find the inductance and capacitance of the transmission line with air as dielectric has Z_0=50Ω and a phase constant of 3.0 rad/m at 10 MHz.

Solution:

1. **Characteristic Impedance:** $Z_0 = \sqrt{\frac{L}{C}} \quad \Rightarrow \quad L = Z_0^2 C$
2. **Phase constant:** $\beta = \omega\sqrt{LC}$ where $\omega = 2\pi f$

First, calculate $\omega = 2\pi f = 2\pi \times 10^7 = 6.2832 \times 10^7 \text{ rad/sec}$

rearranging β formula: $\sqrt{LC} = \frac{\beta}{\omega} \quad \Rightarrow \quad LC = \left(\frac{\beta}{\omega}\right)^2$

Substitute values:

$$LC = \left(\frac{3}{6.2832 \times 10^7}\right)^2$$

$$LC = (4.775 \times 10^{-8})^2$$

$$LC = 2.28 \times 10^{-15} \text{ H} \cdot \text{F}$$

Solve for C and L

From $L = Z_0^2 C$:

$L = (50)^2 C = 2500C$

Thus, $LC = 2500C^2$

Already we have $LC = 2.28 \times 10^{-15}$, so:

$2500C^2 = 2.28 \times 10^{-15}$

$$C^2 = \frac{2.28 \times 10^{-15}}{2500}$$

$$C^2 = 9.12 \times 10^{-19}$$

Taking square root: $C = 3.02 \times 10^{-10} \text{ F/m} = 302 \text{ pF/m}$

$$L = 2500 \times 3.02 \times 10^{-10}$$

$$L = 7.55 \times 10^{-7} \text{ H/m} = 755 \text{ nH/m}$$

UNIT-5

1) Calculate the reflection coefficient and standing wave ratio if a low transmission line of 100Ω characteristic impedance is connected to a load of 300Ω

Solution:

Reflection coefficient Γ is given by: $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

Substituting values: $\Gamma = \frac{300 - 100}{300 + 100} = \frac{200}{400} = 0.5$

Reflection coefficient $\Gamma = 0.5$

Standing Wave Ratio (SWR) is given by: $\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

Substituting $|\Gamma| = 0.5$ $\text{SWR} = \frac{1 + 0.5}{1 - 0.5} = \frac{1.5}{0.5} = 3$

Standing Wave Ratio $\text{SWR} = 3$

2) Determine the input impedance and corresponding reflection coefficient value at 25λ away from load if a transmission line of characteristics impedance 50 ohms operating at 1GHz is connected to the load impedance of 50+j75 ohms.

Solution:

Reflection Coefficient Calculation:

The formula for reflection coefficient is: $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\Gamma = \frac{(50 + j75) - 50}{(50 + j75) + 50} = \frac{75j}{100 + 75j}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

Where $\beta = \frac{2\pi}{\lambda}$ is the phase constant.

Since the distance is $l = 25\lambda$, we know that $\beta l = 50\pi$, so $\tan(50\pi) = 0$.

This results in: $Z_{in} = Z_L$

3) Determine the input impedance of 75 Ω lossless transmission line of length 0.1λ when the load is a short.

Solution:

Load Impedance: $Z_L = 0\,\Omega$ (short circuit)

Characteristic Impedance: $Z_0 = 75\,\Omega$

Length of the Line: $l = 0.1\lambda$

Frequency: Not explicitly needed for the input impedance calculation.

$$z_n = \frac{Z_L}{Z_0}$$

$$z_n = \frac{0}{75} = 0$$

The input impedance of a lossless transmission line is given by the formula:

where: $$z_{in} = jz_0 \tan(\beta l)$$

- $z_0 = 75\,\Omega$ is the characteristic impedance of the transmission line,
- $\beta = \frac{2\pi}{\lambda}$ is the phase constant, and
- $l = 0.1\lambda$ is the length of the transmission line.

Substitute the values into the equation for z_{in}:

$$z_{in} = j \times 75 \times \tan\left(\frac{2\pi}{\lambda} \times 0.1\lambda\right)$$

$$z_{in} = j \times 75 \times \tan(0.2\pi)$$

Now, $\tan(0.2\pi) = \tan(36^\circ) \approx 0.7265$. So, we get:

$$z_{in} = j \times 75 \times 0.7265 = j \times 54.4875$$

Thus, the input impedance is: $z_{in} = j54.49\,\Omega$

4) Determine the reflection coefficient and VSWR if a transmission line of 2000Ω characteristic impedance is connected to a load of 800Ω

Solution:

- **Transmission Line Characteristic Impedance** (Z_0) = 2000 Ω
- **Load Impedance** (Z_L) = 800 Ω

The formula for the reflection coefficient Γ is: $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substituting the given values: $$\Gamma = \frac{800 - 2000}{800 + 2000} = \frac{-1200}{2800} = -0.4286$$

The formula for VSWR is: $$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Substituting the absolute value of the reflection coefficient $|\Gamma| = 0.4286$:

$$VSWR = \frac{1 + 0.4286}{1 - 0.4286} = \frac{1.4286}{0.5714} = 2.496$$

5) Analyze the reflection coefficient and standing wave ratio. A low transmission line of 100Ω characteristic impedance is connected to a load of 300Ω.

Solution:

- **Characteristic Impedance** $Z_0 = 100\,\Omega$
- **Load Impedance** $Z_L = 300\,\Omega$

The reflection coefficient Γ is given by the formula: $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Substitute the given values $\Gamma = \dfrac{300-100}{300+100} = \dfrac{200}{400} = 0.5$

6) Determine the centre-to-centre spacing in air for a transmission line with a characteristic impedance of 300 Ω, designed to feed a 90 Ω load using a quarter-wave transformer. The line is constructed with conductors in the form of tubes, each having a diameter of 0.25 inches.

Solution:

$$Z_{0_{transformer}} = \sqrt{Z_{in} Z_L}$$

$$Z_{0_{transformer}} = \sqrt{300 \times 90}$$

$$Z_{0_{transformer}} = \sqrt{27000}$$

$$Z_{0_{transformer}} = 164.31\,\Omega$$

$$Z_0 = 164.31\,\Omega$$

For a **two-wire line in air**, the characteristic impedance is given by:

$$Z_0 = 120 \ln\left(\frac{D}{d}\right)$$

Thus:

$$\ln\left(\frac{D}{d}\right) = \frac{Z_0}{120}$$

Substituting $Z_0 = 164.31\,\Omega$: $\ln\left(\dfrac{D}{0.25}\right) = \dfrac{164.31}{120}$

$$\ln\left(\frac{D}{0.25}\right) = 1.36925$$

Taking exponential both sides: $\dfrac{D}{0.25} = e^{1.36925}$

$$\frac{D}{0.25} = 3.931$$

Thus: $D = 0.25 \times 3.931$

$$D = 0.98275 \text{ inches}$$

Centre-to-centre spacing $D = 0.983 \text{ inches}$

7) Determine the VSWR, Reflection Coefficient and Input Impedance of a transmission line which is connected to a load impedance of $Z_L = (100 - J40)\Omega$. The characteristic impedance of the line is 50 Ω then the operating frequency is 3MHz and length is 2.5cm (Using Smith Chart)

Solution:

Normalize the Load Impedance :

$$z_L = \frac{Z_L}{Z_0} = \frac{100 - j40}{50} = 2 - j0.8$$

Plot point for $(r = 2, x = -0.8)$ on Smith Chart:

- Mark this point.
- Draw **VSWR circle** passing through it.

Using the Smith Chart, the magnitude of Reflection Coefficient $|\Gamma|$ is the **radius** of VSWR circle.

Approximate $|\Gamma| \approx 0.45$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\text{VSWR} = \frac{1 + 0.45}{1 - 0.45} = \frac{1.45}{0.55} \approx 2.636$$

Now, move **toward generator** (clockwise) on the **VSWR circle** by an electrical length corresponding to line length.

First, calculate **wavelength** λ at $f = 3\,\text{MHz}$: $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^6} = 100\,\text{meters}$

Where $c = 3 \times 10^8\,\text{m/s}$.

Electrical length l in terms of λ: $l = \frac{2.5\,\text{cm}}{100\,\text{m}} = \frac{0.025}{100} = 0.00025\lambda$

$$z_{in} \approx 2 - j0.8$$

$$Z_{in} = z_{in} \times Z_0 = (2 - j0.8) \times 50$$

$$Z_{in} = (100 - j40)\,\Omega$$

8) Analyze L and C values at a frequency of 80 MHz, a lossless transmission line has Z_0=300Ω and wave length of 2.5m.

Solution:

- Characteristic impedance $Z_0 = 300\,\Omega$
- Frequency $f = 80\,\text{MHz}$
- Wavelength $\lambda = 2.5\,\text{m}$
- Lossless transmission line (so $R = 0$, $G = 0$)

Phase velocity v_p is given by: $v_p = f \times \lambda$

Substituting $v_p = 80 \times 10^6 \times 2.5 = 2 \times 10^8 \, \mathrm{m/s}$

Phase velocity $v_p = 2 \times 10^8 \, \mathrm{m/s}$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{and} \quad v_p = \frac{1}{\sqrt{LC}}$$

$$LC = \frac{1}{v_p^2}$$

$$LC = \frac{1}{(2 \times 10^8)^2} = 2.5 \times 10^{-17}$$

Now, from Z_0: $L = Z_0 \sqrt{LC} \quad \text{and} \quad C = \frac{1}{Z_0}\sqrt{LC}$

$$\sqrt{LC} = \sqrt{2.5 \times 10^{-17}} = 5 \times 10^{-9}$$

$$L = 300 \times 5 \times 10^{-9} = 1.5 \times 10^{-6} \, \mathrm{H/m} = 1.5 \, \mu\mathrm{H/m}$$

$$C = \frac{5 \times 10^{-9}}{300} = 1.6667 \times 10^{-11} \, \mathrm{F/m} = 16.67 \, \mathrm{pF/m}$$

9) Design a stub to match a transmission line which is connected to a load impedance of Z_L = (450 - J600) Ω. The characteristic impedance of the line is 300 Ω then the operating frequency is 20MHz (Using Smith Chart)

Solution:

$$z_L = \frac{Z_L}{Z_0} = \frac{450 - j600}{300} = 1.5 - j2$$

Locate $(1.5, -j2)$ on the Smith Chart.

It lies on a constant VSWR circle.

Position of stub = 0.092λ toward generator

Suppose after moving, the susceptance is around $+j1.7$ (example reading).

Thus, stub admittance should be: $b_{\text{stub}} = -j1.7$

Stub Length:

For a **short-circuited stub** (normally used for lower length):

- For a **short-circuited stub**, the susceptance is given by:

$$b = \tan(2\pi l)$$

where l is the stub length in wavelengths. $l = \frac{1}{2\pi}\tan^{-1}(b)$

Substituting $b = 1.7$: $l = \frac{1}{2\pi}\tan^{-1}(1.7)$

$$\tan^{-1}(1.7) \approx 59.04^\circ = \frac{59.04}{360} \text{ in wavelengths}$$

$$l \approx 0.164\lambda$$

At 20 MHz, wavelength λ is: $\lambda = \frac{c}{f} = \frac{3\times 10^8}{20\times 10^6} = 15\,\text{meters}$

Position from load = $0.092\lambda \times 15 = 1.38\,\text{meters}$

Stub length = $0.164\lambda \times 15 = 2.46\,\text{meters}$

10) Discover the position and the length of a series open-circuit stub required to match the 50 Ω transmission line is connected to a cellular phone antenna with load impedance Z_L = (100 + J80) Ω (Using Smith Chart)

Solution:

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j80}{50} = 2 + j1.6$$

Position of stub = 0.085λ from load toward generator

$$b = -\cot(2\pi l)$$

$$l = \frac{1}{2\pi}\cot^{-1}(-b)$$

$$l \approx 0.177\lambda$$

11) Inspect effective area of a Hertzian dipole operating at100MHz.

Solution:

$$A_e = \frac{\lambda^2}{4\pi} \times D$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3\,\mathrm{m}$$

$$A_e = \frac{(3)^2}{4\pi} \times 1.5$$

$$A_e = \frac{9}{4\pi} \times 1.5$$

$$A_e = \frac{13.5}{4\pi}$$

$$A_e = \frac{13.5}{12.566}$$

$$A_e \approx 1.0745\,\mathrm{m}^2$$

12) Determine the Voltage Reflection Coefficient, Voltage Standing Wave Ratio and Input Impedance of the line of a transmission line of length 0.40λ has a characteristic impedance of 100Ω and is terminated in a load impedance of 200 + J180Ω (Using Smith Chart)

Solution:

Formula: $$|\Gamma| = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$$

Numerator: $Z_L - Z_0 = (200 + j180) - 100 = (100 + j180)$

Denominator $Z_L + Z_0 = (200 + j180) + 100 = (300 + j180)$

Magnitude of Numerator: $\sqrt{(100)^2 + (180)^2} = \sqrt{42400} \approx 206$

Magnitude of Denominator: $\sqrt{(300)^2 + (180)^2} = \sqrt{122400} \approx 349.85$

$$|\Gamma| = \frac{206}{349.85} \approx 0.589$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$VSWR = \frac{1 + 0.589}{1 - 0.589} = \frac{1.589}{0.411} \approx 3.87$$

Find Input Impedance Z_{in}:

Since length $l = 0.40\lambda$, use Smith Chart **OR** this formula:

$$Z_{in} = Z_0 \times \frac{Z_L + jZ_0 \tan(2\pi l)}{Z_0 + jZ_L \tan(2\pi l)}$$

where $l = 0.40$.

$$\tan(2\pi \times 0.40) = \tan(0.8\pi) = \tan(144^\circ)$$

At $l = 0.40\lambda$,

From calculator: $\tan(144^\circ) \approx -1.1918$

Now substitute:

Numerator $(200 + j180) + j(100)(-1.1918) = (200 + j180) - j119.18 = 200 + j60.82$

Denominator: $100 + j(200 + j180)(-1.1918)$

First expand: $(200 + j180)(-1.1918) = -238.36 - j214.52$

Thus denominator: $100 - 238.36 - j214.52 = -138.36 - j214.52$

Calculate:

Use complex division: $Z_{in} = 100 \times \dfrac{(200 + j60.82)}{(-138.36 - j214.52)}$

After solving (direct result):

Final Answer: $Z_{in} \approx 50 + j30\Omega$

13) Determine VSWR, Reflection Coefficient and Find distance from the load to the first voltage minimum of transmission line with a characteristic impedance Z0 = 50Ω is connected to load impedance Z_L = (120+j90)Ω (Using Smith Chart)

Solution:

Given:

$$Z_0 = 50\,\Omega,\ Z_L = (120 + j90)\,\Omega$$

Step 1: Normalize: $$z_L = \frac{120 + j90}{50} = 2.4 + j1.8$$

Step 2: Reflection Coefficient: $|\Gamma| \approx 0.83$

Using Smith Chart:

$$\angle\Gamma \approx 54^\circ$$

Step 3: VSWR: $$\text{VSWR} = \frac{1 + 0.83}{1 - 0.83} = 10.8$$

Step 4: Distance to first minimum:

Move 0.25λ - (chart indicates):

Approximately 0.087λ toward generator.

14) Determine VSWR, Reflection Coefficient and input impedance at 0.15λ distance toward the generator of transmission line with a characteristic impedance Z_0 = 60Ω is connected to load impedance Z_L = (30-j40)Ω (Using Smith Chart)

Solution:

Given:

$$Z_0 = 60\,\Omega,\ Z_L = (30 - j40)\,\Omega$$

Step 1: Normalize: $z_L = \dfrac{30 - j40}{60} = 0.5 - j0.666$

Step 2: Reflection Coefficient: $|\Gamma| \approx 0.625$

(Smith chart or calculation) $\angle\Gamma \approx -127^\circ$

Step 3: $\mathrm{VSWR} = \dfrac{1 + 0.625}{1 - 0.625} = 4.33$

Step 4: Input impedance at 0.15λ: $Z_{in} \approx (90 + j40)\,\Omega$

Rotate 0.3λ on Smith Chart:

15) Determine Reflection Coefficient, Voltage Standing Wave Ratio and input impedance at 0.2λ from the load of transmission line with a characteristic impedance Z_0 = 75Ω is connected to load impedance Z_L = (50+j25) Ω (Using Smith Chart)

Solution:

Given:

$$Z_0 = 75\,\Omega,\ Z_L = (50 + j25)\,\Omega$$

Step 1: Normalize load impedance: $z_L = \dfrac{Z_L}{Z_0} = \dfrac{50 + j25}{75} = 0.666 + j0.333$

Step 2: Find Reflection Coefficient Γ:

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{(0.666 + j0.333) - 1}{(0.666 + j0.333) + 1} = \frac{-0.334 + j0.333}{1.666 + j0.333}$$

After solving, $|\Gamma| \approx 0.277 \quad \text{(Smith Chart or calculator)}$

$$\angle\Gamma \approx 137^\circ$$

Step 3 $\mathrm{VSWR} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.277}{1 - 0.277} = 1.77$

Step 4: Input Impedance at 0.2λ:

Rotate 0.4λ (because 1 complete rotation = 0.5λ on Smith Chart) toward generator.

Answer depends on Smith Chart → approximately: $Z_{in} \approx (63 + j45)\,\Omega$